Interpreting Quantifier Combinations
Hintikka's Thesis Revisited

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Universiteit van Amsterdam

Logic, Language, and Reasoning Seminar
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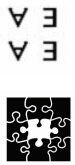
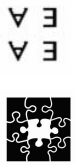


Abstract:

We discuss Hintikka's Thesis that there exist natural language sentences which require non-linear quantification to express their meaning, e.g.:

1. Some relative of each villager and some relative of each townsman hate each other.
2. Some book by every author is referred to in some essay by every critic.

We argue for a novel alternative reading expressible by linear formulae and called conjunctional reading. Our empirical research shows that people tend to interpret H-sentences in a way consistent with it.



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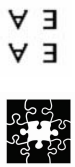

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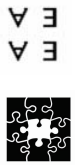
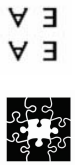


Outline

SENTENCES WE ARE INTERESTED IN

1. **Most girls and most boys hate each other.**
2. One third of girls and half of boys hate each other.
3. 5 girls and 7 boys hate each other.



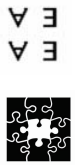
Linear and Branching Reading

Most girls and most boys hate each other.

Linear: $\text{MOST}\ x\ (G(x), \text{MOST}\ y\ (B(y), H(x,y)))$.

Branching: $\begin{matrix}\text{MOST}\ x : G(x) \\ \text{MOST}\ y : B(y)\end{matrix}\ H(x,y)$.

$\exists A \exists A' [\text{MOST}(G, A) \wedge \text{MOST}(B, A') \wedge \forall x \in A \forall y \in A' H(x,y)]$.



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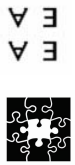

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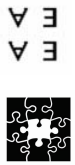

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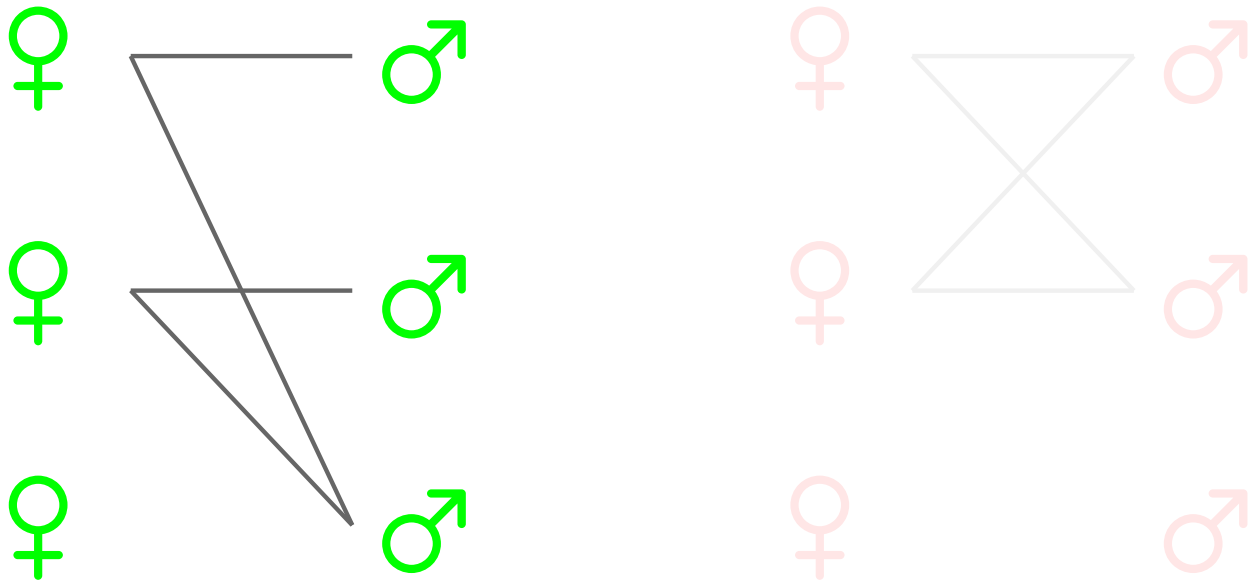

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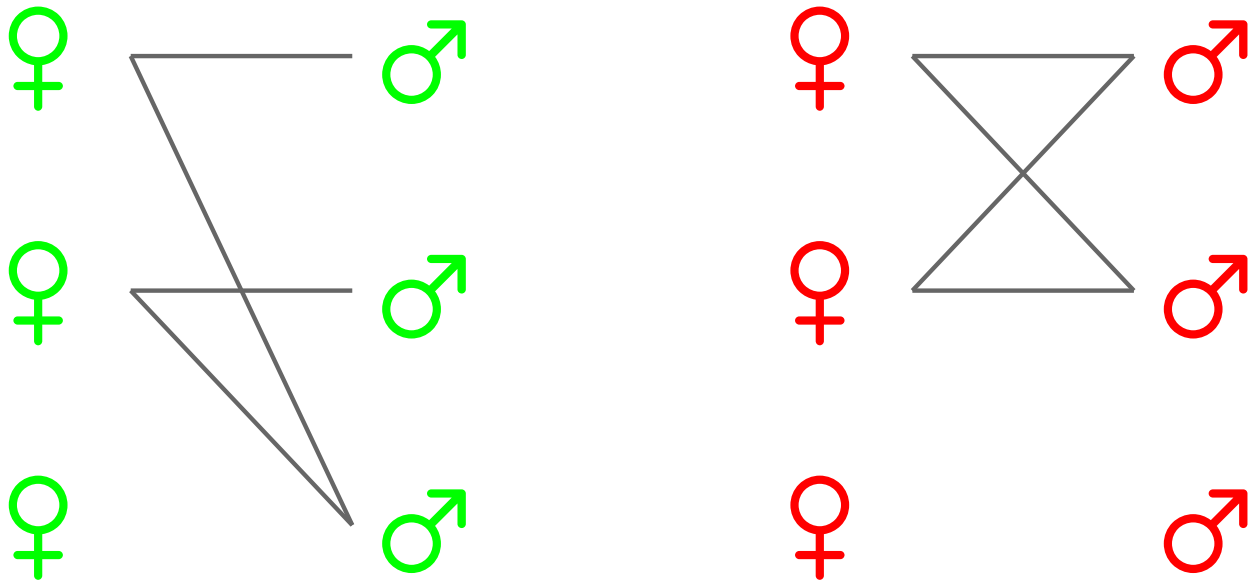

Hintikka's Thesis 1973

Hypothesis

H-sentences have no adequate linear reading.

- E.g., our sentences should be assigned branching reading.
- Provoked lively philosophical and linguistic controversies.

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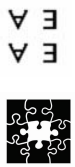

HINTIKKA-LIKE SENTENCES ARE SYMMETRIC

Intuitively equivalent:

1. Most girls and most boys hate each other.
2. Most boys and most girls hate each other.

However,

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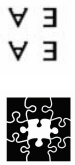

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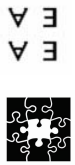

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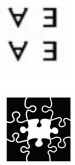

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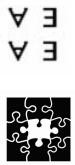

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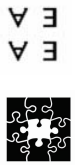

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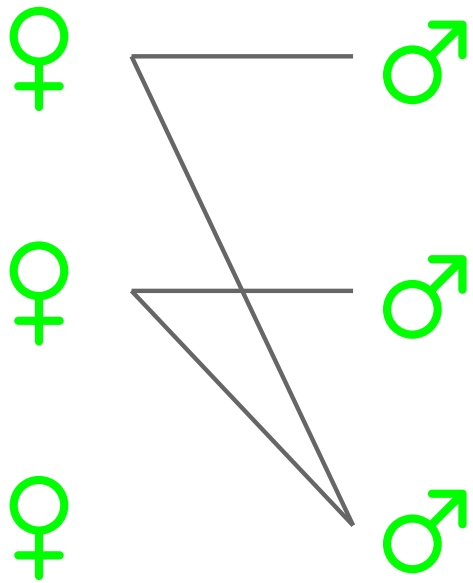

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Therefore, we have to reject the linear reading.

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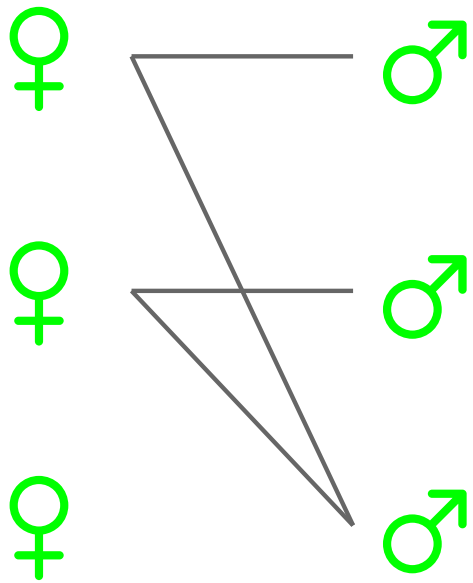

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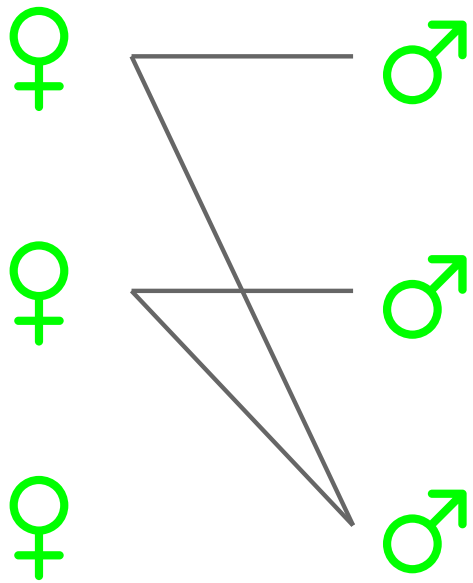

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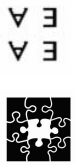


Do we like branching reading?

Branching reading is symmetric, but it is also extremely hard:

Theorem

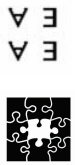
Branching sentences are not expressible in first-order logic enriched by quantifiers occurring in the sentences.

Theorem

Branching sentences are NP-complete.

Question

Do we have any reasonable alternative?



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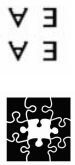

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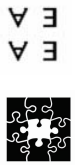

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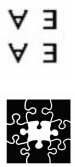

LET US INTRODUCE CONJUNCTIONAL READING

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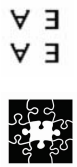

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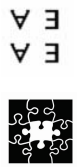

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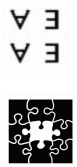

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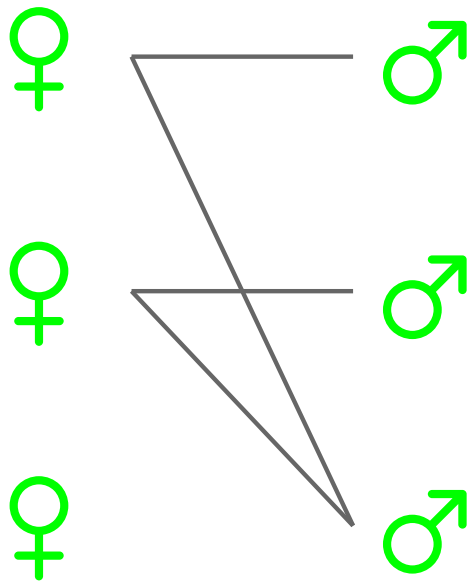

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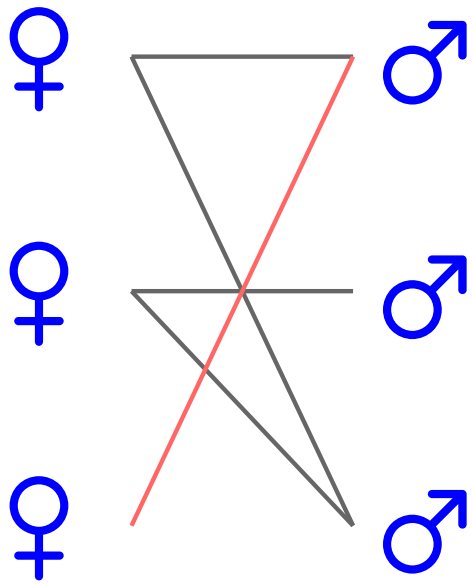
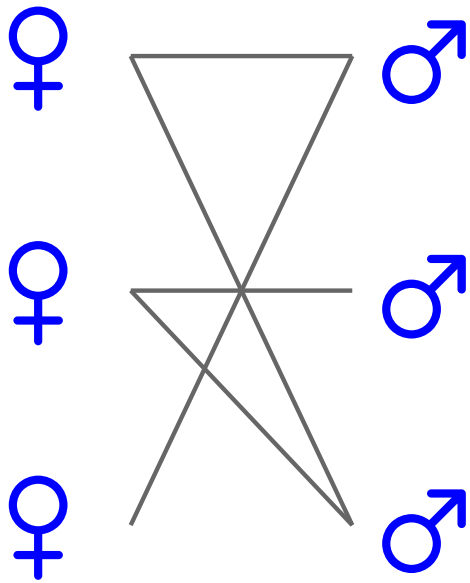
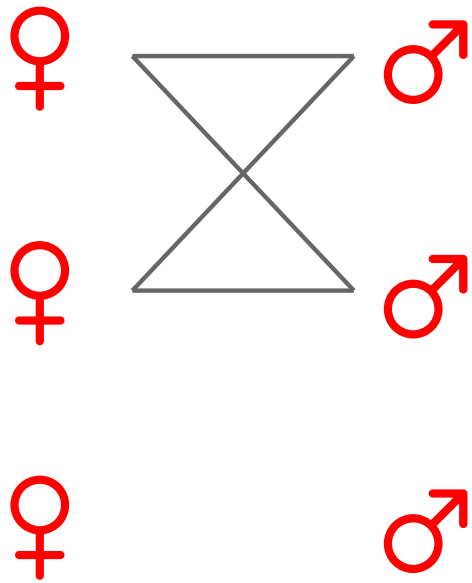


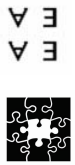
CONJUNCTIONAL VS. BRANCHING READING





Advantages of conjunctional reading

- It is symmetrical.
- It is definable in first-order logic.
- Its logical-value is practically computable.
- It is the strongest reading among weak interpretations.
- It is consistent with representation of reciprocals (Heim et al. 91): `EACH[[QP and QP] [V the other]]`.
- People mostly choose it!



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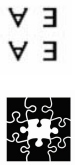

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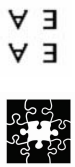

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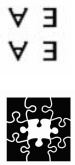

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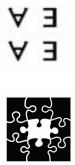

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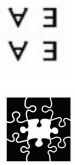

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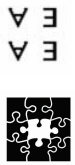

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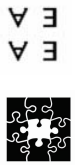

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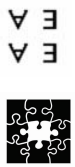


Hypotheses

Hypothesis (1)

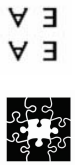
People treat H-sentences as symmetrical sentences.

Hypothesis (2)

People assign to H-sentences conjunctional reading.

Hypothesis (3)

H-sentences are understood in the same way in English and Polish.



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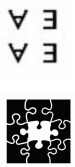
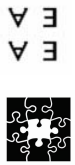


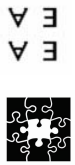
Subjects

- Volunteers.
- 32 native English speakers — undergraduates in computer science, Stanford University.
- 90 native Polish speakers — undergraduates in philosophy, University of Warsaw.
- Many more subjects in previous versions.



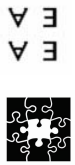
Materials

- Monotonicity of quantifiers influences their difficulty.
- Upward monotone quantifiers easier than downward.
- Only monotone increasing quantifiers “More than *n*”.
- Quantifiers probed shape of geometrical objects.
- The sentences were H-sentences.
- All sentences were authorized by native speakers.
- Pen & paper, no time limit.



STRUCTURE OF THE EXPERIMENT

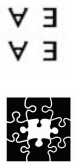
1. Symmetricity test.
2. Branching vs conjunctional interpretation.



DEFINITION

Let Q_1, Q_2 be quantifiers and ψ a quantifier-free formula. We will say that sentence $\varphi := Q_1 x\ Q_2 y\ \psi(x, y)$ is symmetrical if and only if it is equivalent to $\varphi' := Q_2 y\ Q_1 x\ \psi(x, y)$.

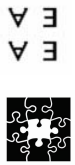
We wanted to check whether subjects treat formulas:
$\varphi := Q_1 x\ Q_2 y\ \psi(x, y)$
$\varphi' := Q_2 y\ Q_1 x\ \psi(x, y)$
as equivalent.



Part I: Symmetricity test

Tasks

- There were 20 tasks.
- 8 valid inference patterns.
- 8 invalid inference patterns.
- 4 simple reasonings with “more than”, “all”, and “some”.
- Non-existing nouns to eliminate pragmatic influence. (e.g. mells, stads, blickets, frobs, . . .)



More than 12 fleems and more than 13 coodles hate each other.

More than 13 coodles and more than 12 fleems hate each other.

VALID NOT VALID

More than 20 wozzles and more than 35 fitches hate each other.

More than 20 fitches and more than 35 wozzles hate each other.

VALID NOT VALID

More than 6 fleems are tulvers.

More than 5 fleems are tulvers.

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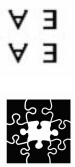
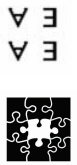


RESULTS

FIRST TEST – SYMMETRICITY

Groups	Polish	American
number of subjects	90	32
all simple correct	45 (50%)	28 (87.5%)
all symmetrical correct	71 (78.89%) (p<0.0001, df=1, $\chi^2 = 30.04$)	29 (90.63%) (p<0.0001, df=1, $\chi^2 = 21.13$)

- Statistical significance in both groups.
- First hypothesis — confirmed.

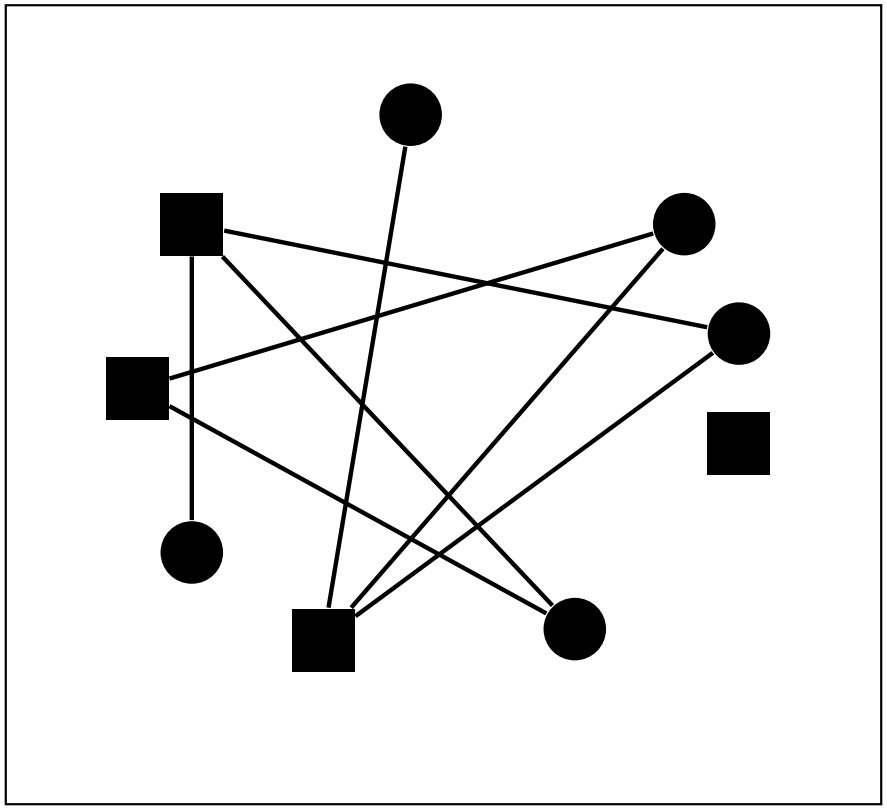


Part II: Branching vs. conjunctional

Tasks

- 9 non-equivalent H-sentences.
- Every sentence paired with a model.
- 7 sentences with a picture satisfying conjunctional reading.
- 2 control tasks – sentences false in pictures.
- Models – B&W pictures.
- Irregularly distributed squares and circles.
- Some objects of different shape connected by lines.
- Number of objects: 9 to 13, and of lines: 3 to 15.

More than 1 square and more than 2 circles are connected by lines.



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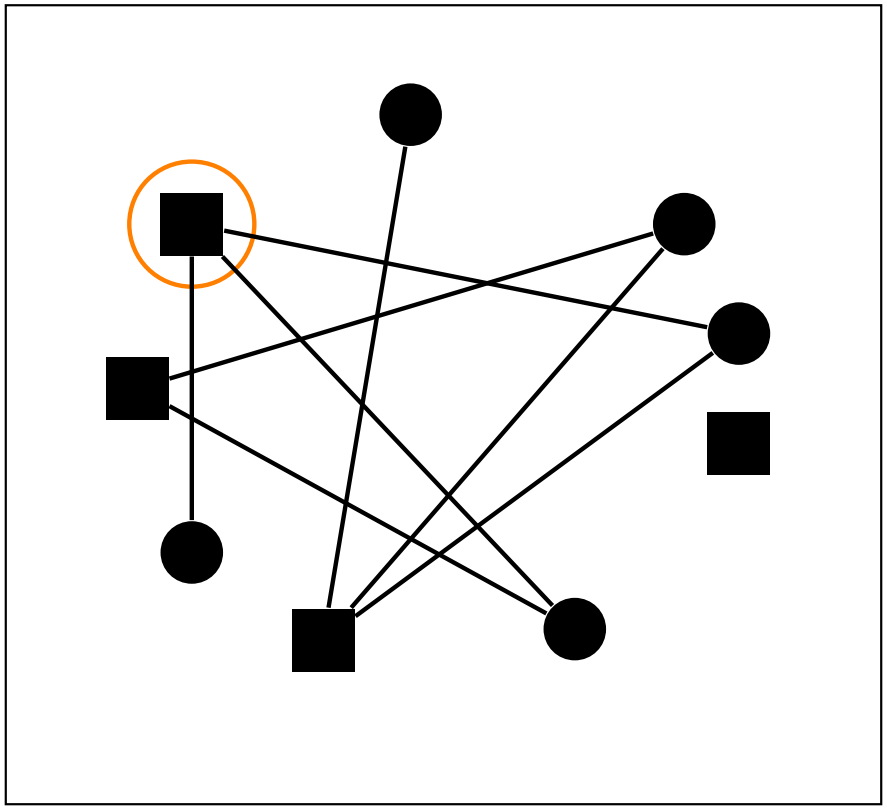

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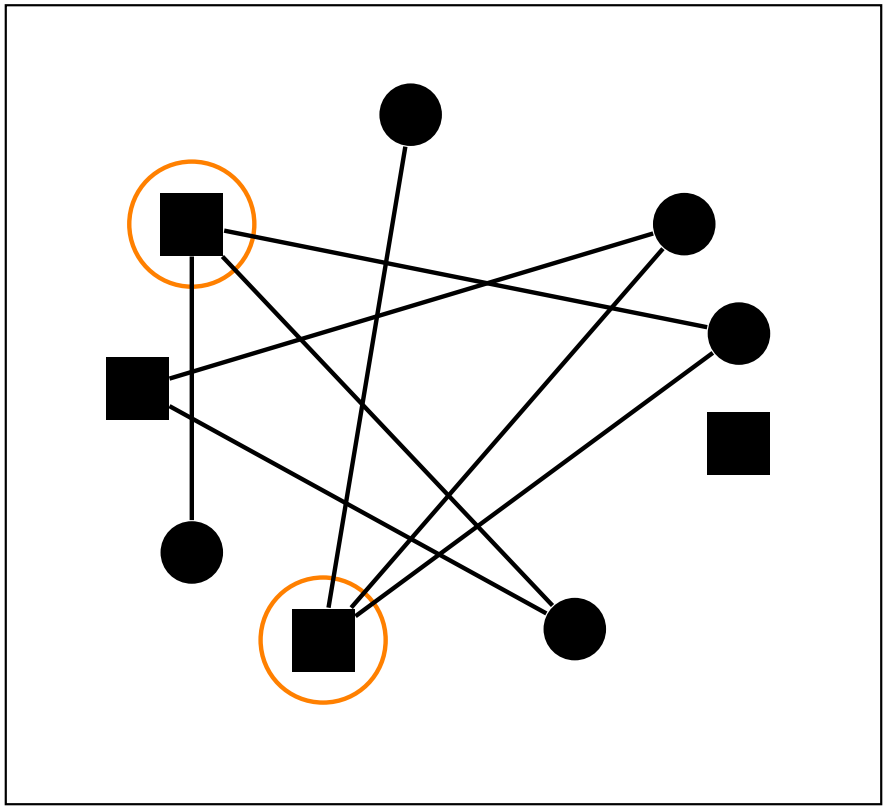

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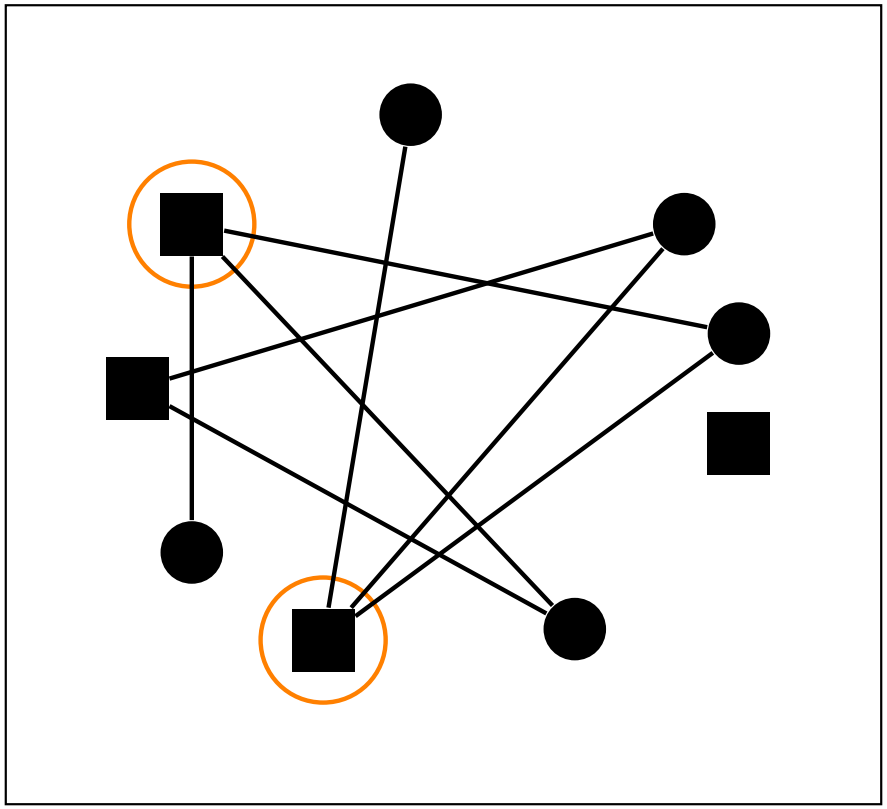

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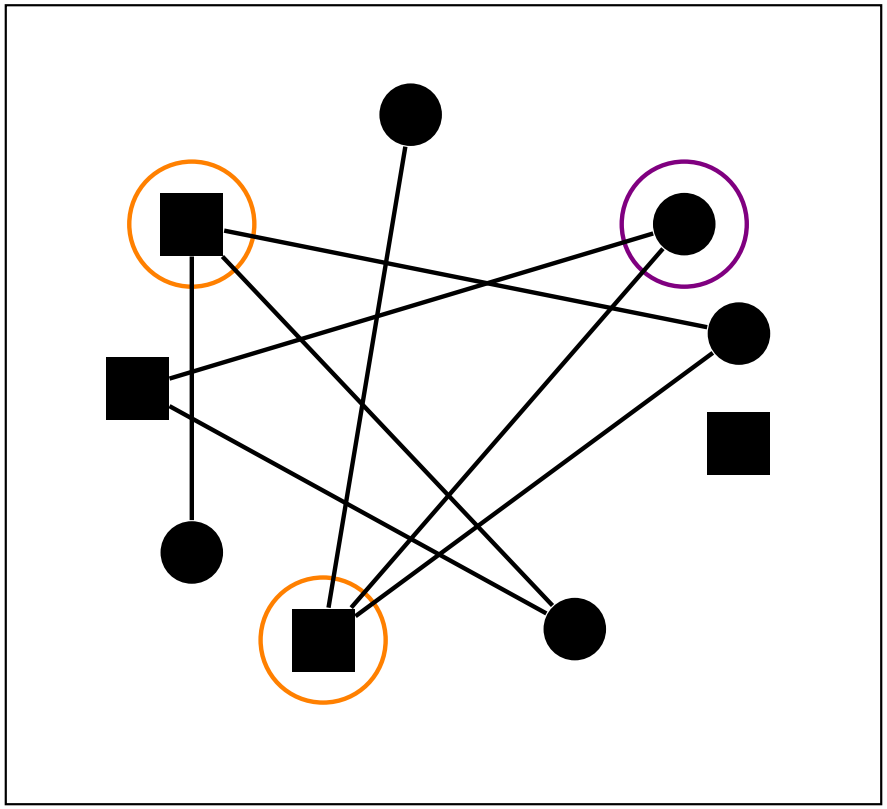

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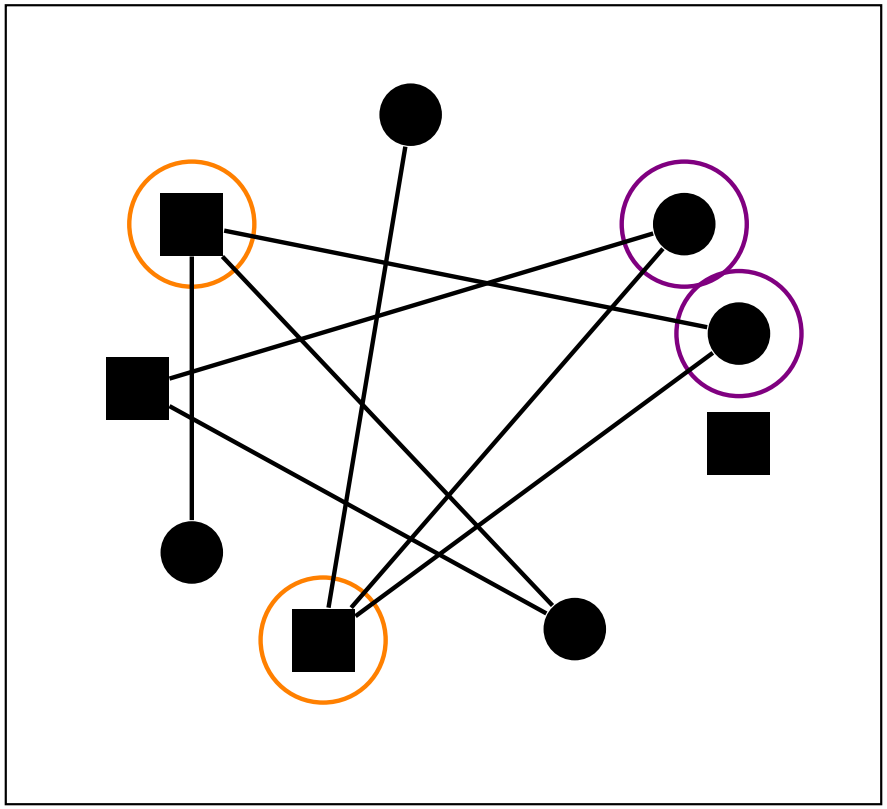

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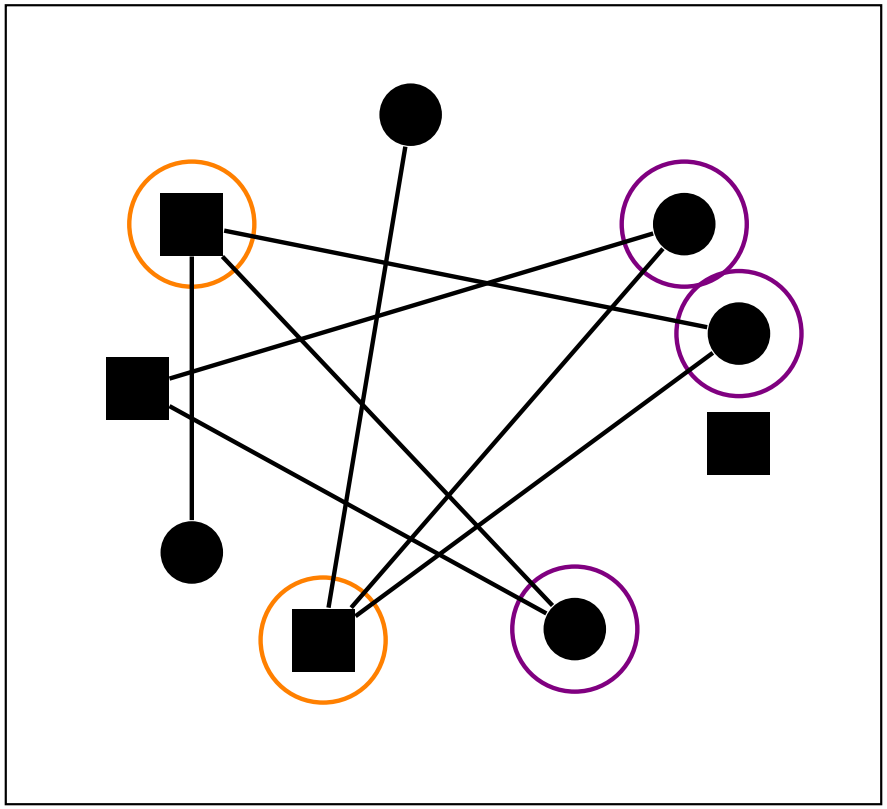
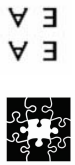


Results

Second test – conjunctional vs branching reading

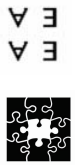
Groups	Polish	American
number of subjects	90	32
most conjunctional	85 (94.4%) (p<0.0001, df=1, $\chi^2 = 71.11$)	31 (96.87%) (p<0.0001, df=1, $\chi^2 = 28.12$)
only conjunctional	67 (74.4%) (p<0.0001, df=1, $\chi^2 = 21.51$)	28 (87.5%) (p<0.0001, df=1, $\chi^2 = 18$)

- Statistical significance in both groups.
- Second hypothesis — confirmed.



THIRD HYPOTHESIS

No statistical differences in reasoning or understanding of H-sentences between English and Polish subjects.



Outline

Results - Checklist

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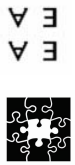

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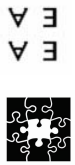

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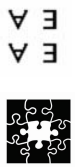

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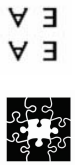

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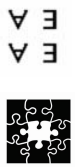
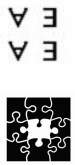


Discussion

1. Why did we choose students with logic background?
2. Why didn't we compare all meanings directly?
3. Why did we omit "each other" in 2nd test?
4. Was it possible to judge sentences by simple counting?



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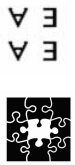

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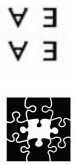

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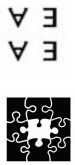


Outline

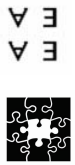
Conclusion

- Although in general ambiguous,
- H-sentences have readings expressible by linear formulae,
- despite what Hintikka and many others claimed.

Our empirical results indicate that:

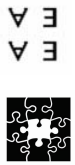
people tend to interpret H-sentences in conjunctional way

at least in empty, experimental, context.



FURTHER RESEARCH

- Find and describe contexts in which H-sentences require non-Fregean analysis.
- Cover other non-Fregean constructions (e.g. reciprocals).



THANK YOU FOR ATTENTION
(Please wake up! It is time for questions!)