Dynamic Epistemic Characterizations for IERS

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Abstract. The iterated regret minimization solution exhibits the good qualitative behavior as that observed in experiments in many games that have proved problematic for Nash Equilibrium(NE). It is worthy exploring epistemic characterizations unearthing players'rationality for an algorithm of Iterated Eliminations Regret-dominated Strategy ($IERS$) related to the solution. In this paper, based on the dynamic epistemic logic (PAL and Plausible Belief Revision Logic), we developed two epistemic regret-game model, characterized the Iterated Elimination Regret-Dominated procedure as a process of dynamic information exchange by taking players'rationality as a proper announcement assertion and a radical upgrade proposition respectively, and proved the related results, thereby threw a new light upon the outcomes of the algorithm $IERS$.

It is well known that deviations between behavior rationality and Bayesian rationality in some games will appear if we set the players' rationality as a precondition of game analysis, and the deviations cause predict outcomes(Nash Equilibriums) are not consistent with empirical observations in these games. Therefore, the study of rational behavior of players, explore the causes of the deviations, and providing reasonable epistemic foundations for the algorithms related to the deviations is always one of the hottest issues in game theory, and has been extended to other subject fields in recent years.(cf.[1] [2] etc.)

In [3] and [4], researchers provided a new iterated algorithm, named Iterated Eliminations Regret-dominated Strategy (shortly,$IERS$), which is one way of trying to capture the intuition that a player wants to do well no matter what the other players do. With the algorithm, firstly, one needs to figure out maximal regret value of players' every strategy according to some rules, given players are uncertain to their opponents' actions, i.e., players think any strategy of their opponents is possible to be chosen, then, to choose strategies corresponding to a minimum regret value after comparing to these maximum regret values, and form a new subgame with the strategies chosen at the previous step, thus, to repeat the process in the new subgame until new subgame no longer change. Halpern et al. called the strategy profiles left in the last suggame, which is no longer change, as new game solutions, Iterated Regret Minimization (here, IR for short). They proved that the new solutions exhibits the same qualitative behavior as that observed in experiments in many famous games that have proved problematic for NE, including Traveler'S Dilemma, the Centipede Game, Nash Bargaining etc, in particularly, the game solution and its algorithm, $IERS$, have particularly appealing when considering inexperienced but intelligent players that play a game for the first time. For example, in Traveler'S Dilemma, given a penalty is 2, minimax regret equilibrium is precisely $(97, 97)$, and agrees well with the experimental results carried out by Becker etc. in [5].

Accordingly, it becomes interesting to explore the epistemic characterization unearthing players' rationality for the algorithm $IERS$. Halpern et al. provided a epistemic characterization for $IERS$ based on an epistemic logic, however, since an epistemic paradox [4]) will arise when they characterized $IERS$ based on a static epistemic logic, so that they had to assigned successively lower probability to higher orders of rationality, and weak a basic premise in game theory, "Rationality is common knowledge among players", by insisting that the higher and higher levels of belief regarding other player's' rationality does not involves common knowledge or common belief. But rationality of common knowledge as a basic premise,it is recorded in almost all of game textbooks, and supported by many game experts and researchers[6][7]. So, under inspiration of [8][9][10], we construct two regret epistemic game models for different dynamic epistemic analysis of the algorithm $IERS$. Based on the epistemic models, We stated, when we describes an iterated elimination dominated procedure as a process of dynamic information exchange by defining players' rationality as a proper announcement assertion or a radical upgrade proposition, the two different interactive epistemic results among players are both line with the outcomes of $IERS$, such that we provide a new characterization to the algorithm $IERS$ in a more intuitive simple way. The characterization can avoid the paradox in the algorithm, as well as it keep well a classic rule in game, that is, it is necessary that rationality should be a common knowledge among players. Moreover, we can construct a uniform frame to analyze and explore rationality in iterated algorithms from players' regret perspective, such as restate the concepts of Weak Rationality (WR) and SR (Strong Rationality) in [9] et ac based on our regret-epistemic game model, thus offer a new perspective to explore logic characterizations for algorithm of Iterated Elimination Strictly Dominated strategies (IESD) and algorithm of Rationalizability corresponding to Bernheims version.

1 Preliminaries

It's only recently that researchers have been looking closely and systematically the research of the iterated minimax regret algorithm and its solution[3][11][4][12]. In fact, the idea of minimax regret was developed (independently) in decision theory by Savage [13]. This approach is to minimize the worst-case regret. The aim of this is to perform as closely as possible to the optimal course. Since the minimax criterion applied here is to the regret rather than to the payoff itself, it is not as pessimistic as the ordinary minimax approach. In this paper, we choose a simple version–finite pure strategies context, to keep the general proposal as simple as possible, and make the dynamic epistemic logic analysis for the algorithm itself to be the key feature. [1]

Definition 1. *Let $G = \langle N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N}\rangle$ be a strategic form game. A regret game of G is a quintuple $G' = \langle N, \{S_i\}_{i\in N}, \{re_i\}_{i\in N}\rangle$ where $\{re_i\}_{i\in N}$ stands for player i's ex-post regret associated with any profile of pure actions (s_i, s_{-i}) as $re_i(s_i, s_{-i}) = max\{u_i(s'_i, s_{-i}), s'_i \in S_i\} - u_i(s_i, s_{-i})$, and let $re_i(s_i) = max\{re_i(s_i, s_{-i}), \forall s_{-i} \in S_i)\}$ states the regret value of choosing s_i for a player i.*

Definition 2. *Given a game $G = \langle N, \{S_i\}_{i\in N}, \{re_i\}_{i\in N}\rangle$, let s_i and s'_i are available strategies for player i, and a set $S'_{-i} \subseteq S_{-i}$, s_i is regret-dominated by s'_i on S'_{-i} if $re_i(s'_i) < re_i(s_i)$. And for a set $S' \subseteq S$, a strategy $s'_i \in S_i$ is unregrettable with respect to S'_i, if no strategy in S'_i regret-dominates s'_i on S'_{-i}. In additional, a regret-dominated strategy s_i is also called regrettable for a player i.*

Definition 3. *The procedure of Iterated Eliminations Regret-dominated Strategies ($IERS$) is as follows. Given a regret game $G' = \langle N, \{S_i\}_{i\in N}, \{re_i\}_{i\in N}\rangle$, let IUD respectively be the set of iterated regret-undominated strategies of the G' recursively defined as follows.*
$IUD = \prod_{i\in N} IUD_i$, where $IUD_i = \bigcap_{m\geqslant 0} IUD_i^m$, with $IUD_i^0 = S_i$ and $RD_i^0 = \{s_i \in IUD_i^0 \mid s_i$ is regrettable with respect to IUD_i^0 in $G'\}$. For $m \geqslant 1, IUD_i^m = IUD_i^{m-1} \setminus RD_i^{m-1}$, where, $RD_i^m = \{s_i \in IUD_i^m \mid s_i$ is regrettable with respect to IUD_i^m in a subgame G'^m $\}^2$.

It is assumed that at each stage all dominated strategies are simultaneously deleted in Definition 2. In contrast to most equilibrium concepts, $IERS$ yields a rectangular set of strategy profiles, i.e., a Cartesian product of sets. This $IERS$ procedure is illustrated in the figure 1.

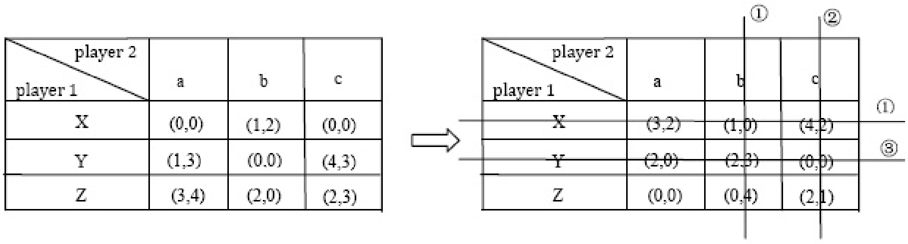

Fig. 1. $IERS$ procedure

$IUD_1^0 = \{X, Y, Z\}, RD_1^0 = \{X\}, IUD_2^0 = \{a, b, c\}, RD_2^0 = \{b\}$
$IUD_1^1 = \{Y, Z\}, RD_1^1 = \varnothing, IUD_2^1 = \{a, c\}, RD_2^1 = \{c\}$
$IUD_1^2 = \{Y, Z\}, RD_1^2 = \{Y\}, IUD_2^2 = \{a\} = IUD_2, RD_2^2 = \varnothing$
$IUD_1^3 = \{Z\} = IUD_1$. Thus, $IUD = \{(Z, a)\}$

[1] Most of our conclusions in this paper can be extended to mixed strategy context.
[2] G'^m is a subgame of G', in which $S_i = IUD_i^m$ and $G'^0 = G'$.

IUD is not consistent with NE in many games, but as we have known that traditional game-theoretic solution concepts-most notably NE-predict outcomes that are not line with empirical observations, this's the main reason that researchers introduce the algorithms of minimax regret into the game theory.

2 Epistemic character of the algorithm of IERS

In order to give a dynamic epistemic analysis of the game solutions as model changes, we provide an epistemic game regret model based on the structure of an original game model.

First, a logic is called a 'regret-game logic' (in short $G' - logic$) if the set of atomic propositions upon which it is built contains atomic propositions of the following forms:
1. Pure strategy symbols $s_i, t_i, ...$: the intended interpretation of s_i is *player i chooses strategy* s_i;
2. Symbols Ra_i^{re}, meaning *player i is rational*, symbols Br_i^* interpreted as *the best response of player i* and a symbol GS meaning *it is a Game Solution with max-minimizing regret algorithm*;
3. Atomic propositions of the form $s_i \succ^1 s_i'$ means the strategy s_i is at most as regrettable as the strategies s_i' for player i, or s_i regret-dominant s_i'.

Next, we define a frame for $G' - logic$ as follows:

Definition 4. *Given a game with regret* G', $\mathfrak{F}_G' = \langle W, \{\sim_i\}_{i\in N}, \{f_i\}_{i\in N}\rangle$ *is a frame of* $G' - logic$ *, where*

- $W \neq \emptyset$*: consists of all players' pure strategy profiles.*
- $\{\sim_i\}$ *is an accessibility relation for player i, which is defined as the equivalence relation of agreement of profiles in the i'th coordinate.*
- $f_i : W \to S_i$ *is a pure strategic function, which satisfies the following property:* $w \sim_i v$ *iff* $f_i(w) = f_i(v)$.

Simply, a frame for $G' - logic$ adds to a Kripke S5 frame a function that associates with every state w a strategy profile $f(w) = (f_1(w), ..., f_m(w)) \in S$. Here, the restriction $w \sim_i v$ iff $f_i(w) = f_i(v)$ is to say that player i know his own choice: if she chooses strategy s_i, then she knows that she chooses s_i. This accords with our intuition. Here, for convenience, we denote $R_i(w) = \{v \mid w \sim_i v, v \in W\}$, and $\|s_i\| = \{w \in W \mid f_i(w) = s_i\}$.

Definition 5. *An epistemic model* $M_{G'}$ *over* $G' - logic$ *is obtained by incorporating the following valuation on a* $\mathfrak{F}_G'$*:*

$$\begin{array}{lll} M_{G'}, w \vDash s_i & \textit{iff} & w \in \|s_i\| \\ M_{G'}, w \vDash (s_i \succcurlyeq^1 s_i') & \textit{iff} & \exists v \in \|s_i'\|), re_i(s_i, f_{-i}(w)) \leq re_i(s_i', f_{-i}(v)) \\ M_{G'}, w \vDash (s_i \succ^1 s_i') & \textit{iff} & \forall v \in \|s_i'\|), re_i(s_i, f_{-i}(w)) < re_i(s_i', f_{-i}(v)) \\ M_{G'}, w \vDash Ra_i^{re} & \textit{iff} & M_{G'}, w \vDash s_i \wedge (\bigwedge_{a \neq s_i}) K_i(s_i \succcurlyeq^1 a). \end{array}$$

According to the definition, our rationality has a straightforward game-theoretic meaning. It says that a rational player always choose the strategies which she knows are at least as good as her others. In details, a player i is rational at a state if she can know what she chooses at the current state is not regret-dominated, that is, the rational players always try to choose an act that minimizes his regret, given she is uncertain about what her opponents will do. It is easy to verify that Ra_i^{re} fails exactly at the rows or the columns with which the regret-dominated strategies correspond for player i in a general epistemic regret-game model $M_{G'}^*$. For instance, in figure 2, Ra_2^{re} fails at the states $(X, b), (Y, b)$ and (Z, b), and Ra_1^{re} fails at the states $(X, a), (X, b)$ and (X, c) of the original model $M_{G'}$.

As we mentioned previously, dynamic analysis of iterated elimination algorithms always has to do with changing of a epistemic model. So, In the following, we call the above epistemic regret-game model $M_{G'}$ a full epistemic regret-game model, and take any submodel of a full epistemic regret-game model $M_{G'}$ as a general epistemic game model $M_{G'}^*$.

If we characterize the minimax algorithm in a static epistemic logic without any dynamic modal operator, a paradox will arise (see [4], so that we have to use some complex methods or techniques to provide a reasonable epistemic foundation of the algorithm, such as assigning successively lower probability to higher orders of rationality, and abandoning or relaxing the most foundational rule in game theory, i.e., the common knowledge in rationality is necessary to form a game solution.

However, if we analyze solution algorithms as processes of learning which change game models, we can not only avoid these drawbacks, and since there are always dynamic intuitions concerning activities of deliberation and communication in a game, we also understand equilibriums or empirical observations in some game better. Therefore, it's more appropriate to deal with assumptions about rationality in dynamic epistemic logic.

3 Dynamic epistemic analysis for *IERS*

3.1 Based on hard information updating

In this subsection, we will describe the procedure of $IERS$ as the process of repeated announcement for the rationality assertion during players deliberate in a one-shot game, and we show "the announcement limit of the rationality assertion" always is consistent with the outcomes by solving a game with the algorithm $IERS$.

First, it is known that the assertions which players publicly announce must be the statements which they know are true in PAL. The following theorems guarantee that the rationality notion in Definition 5 can be as an assertion of a public announcement.

Theorem 1. *Every finite general epistemic regret-game model has worlds with* $Ra^{re}(Ra^{re} = \bigcap_{i\in n} Ra_i^{re})$ *true.* [3]

It follows from the theorem 1 that the rationality is self-fulfilling on finite general epistemic regret-game models. Additionally, we can easily conclude the following result from the semantic interpretations Ra_i^{re} and the properties of model $M_{G'}$.

Theorem 2. *The rationality is epistemically introspective. i.e., the formula* $Ra_i^{re} \to K_i Ra_i^{re}$ *is valid on a general epistemic regret-game model.*

As a result, these theorems guarantee that we can successively remove the worlds at which Ra^{re} does not hold in a model $M^*_{G'}$ after repeated announcing the rationality at some actual world. Although this scenario behind an iterative solution algorithm is virtual, it is significant: we can expect players to announce that they are rational, since they would know it. As van Benthem have shown in [9] "as information from another player may change the current game model, it makes sense to iterate the process, and repeat the assertion Ra^{re} if still true".

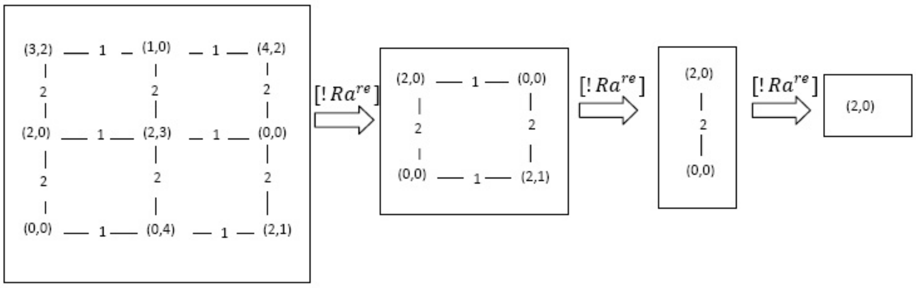

Fig. 2. The public announcement of Ra^{re}

In figure **2**, the left-most model is the model from figure **1**. The other models are obtained by public announcements of Ra^{re} successively for three times. So, in the last submodel, we have: $M_{G'}, (Y, a) \vDash [!Ra^{re}][!Ra^{re}][!Ra^{re}]C_N(GS)$. It indicates that if the players iteratively announce that they are rational, the process of regret-dominated strategies elimination leads them to a solution that is commonly known to be GS.

Theorem 3. *Given a full epistemic game model based on a finite strategic-form G' with regret and an arbitrary world w, w is in a general epistemic game model $M^*_{G'}$ which is stable by repeated announcements of Ra^{re} in the $M_{G'}$ for all players if and only if $f(w) \in IUD$. That is to say,* $w \in \sharp(Ra^{re}, M_{G'}) \Leftrightarrow f(w) \in IUD$.

[3] See the Appendix for proofs of all the theorems in the paper

3.2 Based on soft information updating

Alternatively, we can represent the procedure of $IERS$ algorithm as the process of repeated $soft$ announcement of the rationality among players.[4] When this rationality assertion is believed to be true by every player: it is common belief that everybody believes that each of player is a the rational agent, the solution of one-shot games is just outcome of the $IERS$ algorithm.

To do this, firstly, we need to introduce some new concepts into $G' - logic$.

Theorem 4. *Repeated truthful radical upgrade* $\Uparrow \boldsymbol{P}$ *in epistemic-doxastic logic stabilizes every model (with respect to which it is correct).*[5].

Next, we redefine a frame $\mathfrak{F}'_{G'}$ of $G' - logic$ and a epistemic-doxastic regret-game model $M'_{G'}$ based on a given game G' with regret. In fact, we can provide the frame $\mathfrak{F}'_{G'}$ just adding a plausibility relation (we mentioned in the section 3) for every player i to the frame $\mathfrak{F}_{G'}$ Meanwhile, let $R'_i(w) = Min_{\preceq_i}([w]_i)$, and $\|s_i\|' = R'_i(w) \cap \|s_i\|$. We introduce the semantic interpretation for those game propositions in $M'_{G'}$ as follows,

Definition 6. *An epistemic model* $M'_{G'}$ *over* $G' - logic$ *is obtained by incorporating the following valuation on a* $\mathfrak{F}''_G$:

$M'_{G'}, w \vDash s_i \quad$ *iff* $\quad f_i(w) = s_i;$
$M'_{G'}, w \vDash (s_i \succsim s'_i) \quad$ *iff* $\quad \exists v \in \|s'_i\|'), re_i(s_i, f_{-i}(w)) \leq re_i(s'_i, f_{-i}(v))$
$M_{G'}, w \vDash (s_i \succ' s'_i) \quad$ *iff* $\quad \forall v \in \|s'_i\|'), re_i(s_i, f_{-i}(w)) < re_i(s'_i, f_{-i}(v))$
$M'_{G'}, w \vDash Ra_i^{re'} \quad$ *iff* $\quad M_{G'}, w \vDash s_i \wedge (\bigwedge_{a \neq s_i} B_i(s_i \succsim a))$.

Accordingly, by the definitions of radical upgrade in [16], it is easy to justify that the radical upgrade stream $\Uparrow \boldsymbol{Ra^{re'}}$ is truthful, since it is reasonable that we take one of the worlds where GS is true as a actual world, and $Ra^{re'}$ always holds at the world.

Corollary 1. *Repeated truthful radical upgrade* $\Uparrow \boldsymbol{Ra^{re'}}$ *in epistemic-doxastic logic stabilizes every model (with respect to which it is correct).*

And similar to the Definition 6, we can define,

Definition 7. *For any epistemic model* $M'_{G'}$ *and formula* φ*, the radical upgrade stabilization* $\#(\Uparrow \varphi, M'_{G'})$ *is the first model in a repeated upgrade stream where upgrade* φ *has no further effect, and* $W^{\#(\Uparrow\varphi, M'_{G'})}$ *is the set of possible worlds which agents considers the most likely after repeated upgrade* φ*, i.e.,* $W^{\#(\Uparrow\varphi, M'_{G'})} = \{w \in Min_{\preceq_{i \in N}}(W) \mid w \vDash \varphi\}$*, and call it as a kernel of the* $\#(\Uparrow \varphi, M'_{G'})$.

It is illustrated in the figure 2, how a radical upgrade $\Uparrow Ra^{re'}$ upgrades the regret-game illustrated in figure 1. Finally, we also show another characterization theorem for $IERS$.

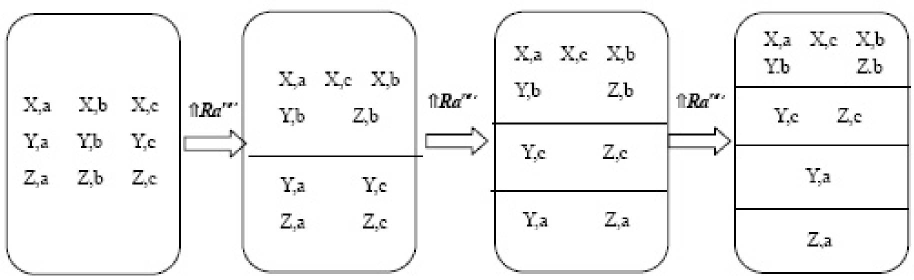

Fig. 3. The radical update of $Ra^{re'}$

Theorem 5. *Given a full epistemic-doxastic game model* $M'_{G'}$ *based on a regret-game* G' *and an arbitrary world* w*,* $w \in W^{\#(\Uparrow Ra^{re'}, M'_{G'})}$ *if and only if* $f(w) \in IUD$.

[4] The soft announcement refers to a radical update introduced in [14].
[5] The proof of this theorem is found in [15]

4 Related Approaches

In in [4], Halpern and Pass put forward a new game solution(they called an iterated regret minimization, similar to the regret equilibrium) , and stated the rationality and significance of the game solution by many examples from the game theory. Meanwhile, they also provided the epistemic characterization for the algorithm $IERS$ (or say iterated regret minimization solution) using Kripke structure similar to the way we did, they defined the atomic proposition, "a player i is rational at a world w in a epistemic game model (denoted by RAT_i", as her current strategy is a best response to strategy sequences $\langle s(B_i^0(w)), s(B_i^1(w)), ...\rangle$ (where $B_i^0(w)$ consists of the worlds that i considers most likely at w, and the worlds in $B_i^1(w)$ are less likely, and so on), i.e., $M, w \vDash RAT_i$ if $s_i(w)$ is a best response to the strategy sequence $\langle s(B_i^0(w)), s(B_i^1(w)), ...\rangle$, and they proved, this game solutions resulted from the algorithm $IERS$ involves higher and higher levels of belief regarding other players' rationality. At this point, we have the same viewpoint as theirs we describe a iterated elimination dominated procedure as a process of dynamic information exchange in the dynamic epistemic logic (PAL or Plausible Belief Revision Logic), it is natural that these higher levels of belief regarding other players' rationality become an implicit requirement for players' belief. The implication is derived from the essential prosperities of these dynamic logic, for example, after public announcing a formula φ in PAL, player i can delete the worlds in her mind which are not satisfied the formula φ, in other words, she never reconsider the worlds as epistemic possible worlds for her (Similar scenario will happen in the belief revision with the radical upgrade $\Uparrow Ra^{re'}$, since i thinks the Ra^{re}-worlds become better than all the $\neg Ra^{re'}$, and keep the ordering at the later upgrade). Thus, she just consider her strategy based on those worlds satisfied Ra^{re} (or the worlds in her the best plausible area). It implies that player i's final choice must be based on the higher and higher the knowledge of (or the belief of) other players' rationality. Nevertheless, in order to avoid a paradox similar to the paradox in the Iterated Elimination of weakly dominated strategy[6], they also insisted that the higher and higher levels of belief regarding other players' rationality does not involves common knowledge or common belief, rather, higher levels of beliefs are accorded lower levels of likelihood, i.e., they assigned successively lower probability to higher orders of rationality. Considering this paradox doesn't arise in our way for the essential prosperities of our dynamic logic again, and the rationality defined by us is self-fulfilling, so, we keep well the classic rule in game theory,that is, it is necessary for analyze a game that rationality is common knowledge among players. Therefore, our dynamic analysis for $IERS$ is more appealing, and it may be more suitable to be extended to Dynamic Model Checking in computer science.[7]

Additionally, there is also a large amount of literatures on the algorithms of iterated elimination either in the field of logic, computer science, or of game theory. [8] In particular, [9] describe and characterize different algorithms in game theory by redefining rationalities based on epistemic logic. Our intellectual debt towards [9] is clear. Compared to their work, we extend their findings in some sense. In fact, we can also restate their results based on our epistemic regret-game frame, provide a new kind of epistemic characterization for the algorithms which have been studied by them. For example, [9] defined two types of rationality, the weak rationality and the strong rationality, which are denoted by WR_i and SR_i. Here, we redefine these rationality assertion on the epistemic regret-game frame as follows,

$$M_{G'}, w \vDash (s_i \succcurlyeq^2 s_i') \quad \text{iff} \quad (re_i(s_i, f_{-i}(w)) \leq re_i(s_i'(v), f_{-i}(w)))$$
$$M_{G'}, w \vDash (s_i \succ^2 s_i') \quad \text{iff} \quad (re_i(s_i, f_{-i}(w)) < re_i(s_i'(v), f_{-i}(w)))$$
$$M_{G'}, w \vDash WR_i' \quad \text{iff} \quad (M_G, w) \models s_i \wedge (\bigwedge_{a \neq s_i} \langle K_i \rangle (f_i(w) \succcurlyeq^2 a))$$
$$M_{G'}, w \vDash SR_i' \quad \text{iff} \quad (M_G, w) \models s_i \wedge \langle K_i \rangle (\bigwedge_{a \neq s_i} (f_i(w) \succcurlyeq^2 a)))$$

Thus, a weak rational player i thinks it is possible that the regret raised by the current strategy is not greater her other strategies, e.g., she can know that there is no alternative action which she knows to reduce her regret, and a strong rational player i thinks it's possible that the current strategy doesn't make her regret more. In other words, a player with strong rationality always a bit optimistic

Based the uniform structure, one can analyze and explore rationality implied iterated algorithms from playersregret perspective, also she can compare the strength of these rationality and the of stable models

[6] cf. [4]

[7] Some of our dynamic epistemic analysis for iterated elimination algorithms in the game theory have been extend in the field of Dynamic Model Checking, cf.[17]

[8] cf. [10][18][19][20] etc.

Theorem 6. *The rationality Ra^{re} is stronger than the rationality WR', i.e., $Ra^{re} \to WR'$, and but not vise versa.*

Corollary 2. $\sharp(Ra^{re}, M_{G'}) \subsetneqq \sharp(WR', M'_{G'})$

However, there is no relation between Ra^{re} and SR'. For instance, in the game G3 is from [8],Ra_2^{re} holds at the worlds :
$(A, a), (B, a), (C, a), (A, c), (B, c), (C, c)$, but SR_2' is true at the worlds:
$(A, a), (B, a), (C, a), (A, b), (B, b), (C, b)$.

player 1 \ player 2	a	b	c
A	(2,3)	(1,0)	(1,1)
B	(0,0)	(4,2)	(1,1)
C	(3,1)	(1,2)	(2,1)

G3

player 1 \ player 2	a	b	c
A	(1,0)	(3,3)	(1,2)
B	(3,2)	(0,0)	(1,1)
C	(0,1)	(3,0)	(0,1)

G'3

Fig. 4. comparing Ra^{re} to SR'

5 Conclusion and Further Direction

What the paper showed: iterated hard and soft update is a powerful method that also applies to non-standard game solution algorithms such as minimizing regret. This brings more kinds of recent work in the foundations of game theory within the scope of dynamic-epistemic logic.But now it is time to go beyond proving such single results. Here are a few directions that we intend to pursue:

Introducing logics for qualitative reasoning We will introduce modal logics over matrix games in the spirit of [19] referring to agents available actions, knowledge and preferences with propositional constants for positions of rationality and/or regret, and study the qualitative calculus of reasoning about these notions in interactive behavior.

Comparing, combining, and reducing methods Comparing methods like $IESD$ and $IERS$, we see that one may be better than another depending on the structure of the given game. We will investigate what happens when agents have a variety of such methods available. One possibility is that one method may simulate another, by means of translating the given game systematically into one with changed outcome values. Moreover, there are games where both methods make sense intuitively. We will start with sequential combinations of solution methods, starting from very concrete questions such as whether $IESD; IERS = IESD; IERS$? The eventual goal would be an algebra of solution methods.

Linking up with limit behavior in learning theory We have only considered cases where games get solved through iterated soft updates with regret statements. But many other scenarios can have the same features, including infinite sequences where the approximation behavior itself is the focus of interest. In particular, we are interested in connecting our setting with the learning-theoretic scenarios and extended temporal update logics suggested by the results of [21];[22]

References

1. Bernheim, D.: Rationalizable strategic behavior. Econometrica **52**(4) (1984) 1007–1028
2. David, P.: Rationalizable strategic behavior and the problem of perfection. Econometrica **52**(4) (1984) 1029–1050
3. Renou, L., Schlag, K.H.: Implementation in minimax regret equilibrium. Games and Economic Behavior **71**(2) (2011) 527–533
4. Halpern, J.Y., Pass, R.: Iterated regret minimization: A new solution concept. Games and Economic Behavior **74**(1) (2012) 184–207
5. Becker, T., Carter, M., Naeve, J.: Experts playing the travelers dilemma. Discussion paper (2005) 252.
6. Aumann, R.J.: Rationality and bounded rationality. Games and Economic Behavior **53** (1999) 293–291

7. Rubinstein, A.: A Course in Game Theory. The MIT Press, Cambridge, Mass. (1994)
8. van Benthem, J.: Logical Dynamcis of Information. Cambridge university press (2011)
9. van Benthem, J.: Rational dynamics and epistemic logic in games. International Game Theory Review **9**(1) (2007) 13–45
10. Bonanno, G.: A syntactic approach to rationality in games with ordinal payoffs. Volume 3. Amsterdam: Amsterdam University Press (2008)
11. Renou, L., Schlag, K.H.: Minimax regret and strategic uncertainty. Journal of Economic Theory **145**(1) (January 2010) 264C286
12. Stoye, J.: Axioms for minimax regret choice correspondences. Journal of Economic Theory **146**(6) (2011) 2226–2251
13. L.J.Savage: The theory of statistical decision. J.Amer.Statist.Ass **46** (1951) 55–67
14. van Benthem, J.: Dynamic logic for belief revision. Journal of Applied Non-Classical Logics **14**(2) (2004) 129–155
15. Baltag, A., Smets, S.: Group belief dynamics under iterated revision: fixed points and cycles of joint upgrades. In: Proceedings of Theoretical Aspects of Rationality and Knowledge. (2009) 41–50
16. Baltag, A., Smets, S.: Group belief dynamics under iterated revision: fixed points and cycles of joint upgrades. In: Proceedings of the 12th Conference on Theoretical Aspects of Rationality and Knowledge. TARK '09, New York, NY, USA, ACM (2009) 41–50
17. Cui, J., Tang, X.: A method for solving nash equilibria of games based on public announcement logic. SCIENCE CHINA (Information Science) **53**(7) (2010) 1358–1368
18. Halpern, J.Y., Pass, R.: A logical characterization of iterated admissibility. CoRR **abs/0906.4326** (2009)
19. van Benthem, J., Pacuit, E., Roy, O.: Toward a theory of play: A logical perspective on games and interaction. Games **2**(1) (2011) 52–86
20. Gilli, M.: Iterated admissibility as solution concept in game theory. Technical report, University of Milano-Bicocca, Department of Economics (2002)
21. Baltag, A., Gierasimczuk, N., Smets, S.: Belief revision as a truth-tracking process. In: TARK. (2011) 187–190
22. Pacuit, E., Roy, O.: A dynamic analysis of interactive rationality. In: Proceedings of the Third international conference on Logic, rationality, and interaction. (2011) 244–257

A Proof

We provide proofs of all the results not proved in the main text here.

Theorem 1 Every finite general epistemic regret-game model has worlds with $Ra^{re}(Ra^{re} = \bigcap_{i\in n} Ra_i^{re})$ true.

Proof. According to the fact that atomic proposition Ra_i^{re} fails exactly at the rows or columns with which regret-dominated strategies correspond for a player i in a general game model. Consider any general game model $M^*_{G'}$. If there is not a regret-dominated action for all players in $M^*_{G'}$, then Ra^{re} is true at all the worlds in it. Thus, the iterated announcement of Ra^{re} can no more change the game model and get stuck in cycles in this situation. If there is a regret-dominated action for some player in the game, because of the relativity of the definition of regret-dominated strategy, he must have a strategy which is better than this strategy, i.e., if player i has a regret-dominated strategy a, then he must have a strategy which is better than strategy a, say a strategy b. Thus, Ra_i^{re} holds at all the worlds which belong to the row or the column corresponding to the strategy b. On the other hand, for player j, if he has no weakly dominated action, then also Ra_j^{re} holds at all the worlds. Furthermore, Ra_j^{re} holds at the worlds which belong to the row or the column corresponding to strategy b. So, Ra^{re} holds in the general game model. But if player j has also a regret-dominated action, accordingly he must have a dominant action, say action Y, and Ra_j^{re} is true at at the worlds which belong to the row or the column corresponding to the strategy Y. Therefore, Ra^{re} is satisfied at the world (Y, b).

To sum up the above arguments, every finite general game model has worlds with Ra^{re} true.

Theorem 2 The rationality is epistemically introspective. i.e.,
the formula $Ra_i^{re} \to K_i Ra_i^{re}$ is valid on a general epistemic regret-game model.

Proof. Consider a general epistemic regret-game model $M^*_{G'}$ and an arbitrary w in $M^*_{G'}$ such that $M^*_{G'}, w \vDash Ra_i^{re}$ but $M^*_{G'}, w \nvDash K_i Ra_i^{re}$. Because $M^*_{G'}, w \nvDash K_i Ra_i^{re}$, it holds that $\exists v \in R_i(w)$ and $M^*_{G'}, v \nvDash Ra_i^{re}$. According to Definition 10, we have $f_i(v)$ is a regret-dominated action for i by some her actions. And, by the property of the function f_i: $f_i(w) = f_i(v)$ iff $v \in Ra_i^{re}(w)$, we can conclude that $f_i(w)$ is also a regret-dominated action for i, further, $M^*_{G'}, w \nvDash Ra_i^{re}$, contrast to the precondition, i.e., $M^*_{G'}, w \vDash Ra_i^{re}$. So, the formula $Ra_i^{re} \to K_i Ra_i^{re}$ is valid on a general game model.

Theorem 3 Given a full epistemic game model based on a finite strategic-form G' with regret and an arbitrary world w, w is in a general epistemic game model $M^*_{G'}$ which is stable by repeated announcements of Ra^{re} in the $M_{G'}$ for all players if and only if $f(w) \in IUD$. That is to say, $w \in \sharp(Ra^{re}, M_{G'}) \Leftrightarrow f(w) \in IUD$.

Proof. (a) From left to right: if $w \in \sharp(Ra, M_{G'})$, that is to say, $w \in M^*_{G'}$, then $M^*_{G'}, w \vDash Ra$, i.e., $M^*_{G'}, w \vDash \wedge_{i\in N} Ra_i^{re}$.

First we show that: for $\forall i \in N, f_i(w) \notin RD_i^0$. Suppose not. Then $\exists i \in N$, such that $f_i(w) \in RD_i^0$, that is, $f_i(w)$ of player i is regret-dominated in G' by some other strategy $s'_i \in S_i = IUD_i^0$, it means: $re_i(f_i(w)) > re_i(s'_i)$, i.e., $max\{re_i(f_i(w), s_{-i}), \forall s_{-i} \in S_{-i}\} > max\{re_i(s'_i, s_{-i}), \forall s_{-i} \in S_{-i}\}$. Thus, let some $s'_{-i} \in S_{-i}$ satisfied $re_i(f_i(w), s'_{-i}) = max\{re_i(f_i(w), s_{-i}), \forall s_{-i} \in S_{-i}\}$, and $s''_{-i} \in S_{-i}$ satisfied $re_i(s'_i, s''_{-i} = max\{re_i(s'_i, s_{-i}), \forall s_{-i} \in S_{-i}\}$, so, we have $re_i(f_i(w), s'_{-i}) > re_i(s'_i, s''_{-i})$. Accordingly, there exist a $v' \in R_i(w) \cap \|s'_{-i}\|$ and a $v'' \in R_i(w) \cap \|s''_{-i}\|$, satisfied $re_i(f_i(w), f_{-i}(v')) > re_i(f_i(w), f_{-i}(v''))$, considering $re_i(f_i(w), f_{-i}(v'')) \geq re_i(f_i(w), f_{-i}(v))$, where $\forall v \in \|s_i\|$, thereby, $re_i(f_i(w), f_{-i}(v')) > re_i(f_i(w), f_{-i}(v))$, where $\forall v \in \|s_i\|$. In terms of the Definition 6, we can conclude that $M^*_{G'}, w \nvDash Ra_i^{re}$, which contradicting the hypothesis that $M^*_{G'}, w \vDash \wedge_{i\in N} Ra_i^{re}$. Since, for every $w \in W, f_i(w) \in IUD_i^0 = S_i$, it follows that $f_i(w) \in IUD_i^0 \setminus RD_i^0 = IUD_i^1$.

Next we prove the inductive step. Fix an integer $m \geq 1$ and suppose that, for every player $j \in N$, $f_j(w) \in IUD_j^m$, we want to show that, for every player j, $f_j(w) \notin RD_j^m$. Suppose not. Then there exists a player i, satisfied that $f_i(w) \in RD_i^m$, that is, $f_i(w)$ is a regret-dominated in G'^m by some other strategy $s'_i \in IUD_i^m$. Then, $max\{re_i(f_i(w), s_{-i}), \forall s_{-i} \in IUD_{-i}^m\} > max\{re_i(s'_i, s_{-i}), \forall s_{-i} \in IUD_{-i}^m\}$. Since, by hypothesis, for $\forall j \in N, f_j(w) \in IUD_j^m$, it follows-since the prosperity of $f_i(w)$, that's, $v \in R_i(w) \Leftrightarrow f_i(w) = f_i(v)$ that for $\forall v \in R_i(w), f_i(v) \in IUD_i^m$, further, we have
$max\{re_i(f_i(w), f_{-i}(v), v \in R_i(w)\} > max\{re_i(f_i(w'), f_{-i}(v)), v \in R_i(w')\}$, where $w' \in R_i(w) \cap \|s'_{-i}\|$. Thus, similar to the reason above, we can conclude $M^*_{G'^m}, w \nvDash Ra_i^{re}$, again contradicting the fact that $M^*_{G'}, w \vDash Ra_i^{re}$ since $M^*_{G'}$ is a submodel of $M^*_{G'^m}$ and the prosperities of $M^*_{G'}$. So, for every player i, $f_i(w) \in IUD_i^m \setminus RD_i^m = IUD_i^{m+1}$. By induction, $f_i(w) \in IUD_i$ for $\forall i \in N$.

(b) From right to left: Let $f(w) \in IUD = \bigcap_{m \geqslant 0} IUD^m$, by Definition 4, $\forall i \in N\, f(w)$ is never regret dominated IUD^m. So, it means, after m rounds of public announcement Ra^{re}, $M^*_{G'^m}, w \vDash Ra^{re}$, where $M^*_{G'^m}$ is a general epistemic model related to submodel G'^m. Therefore, in terms of the arbitrary of m and the definition 6, it is obvious that $w \in \sharp(Ra^{re}, M_{G'})$

Theorem 5 Given a full epistemic-doxastic game model $M'_{G'}$ based on a regret-game G' and an arbitrary world w, $w \in W^{\#(\Uparrow Ra^{re'}, M'_{G'})}$ if and only if $f(w) \in IUD$.

Proof. (a) From left to right: the proof is similar to the induction proof of Theorem 3 and left to the reader.

(b) From right to left: suppose $f(w) \in IUD = \bigcap_{m \geqslant 0} IUD^m$, but $w \notin W^{\#(\Uparrow Ra^{re'}, M'_{G'})}$, then, either $\#(\Uparrow Ra^{re'}, M'_{G'}), w \nvDash Ra^{re'}$ or $w \notin Min_{\preceq_{i \forall i \in N}}(W)$: if $\#(\Uparrow Ra^{re'}, M'_{G'}), w \nvDash Ra^{re'}$, We might as well let $\#(\Uparrow Ra^{re'}, M'_{G'}), w \nvDash Ra_i^{re'}$, then $f_i(w)$ is a regret-dominated strategy by some strategy for player i result from the semantic definition of $Ra_i^{re'}$, thus, $f_i(w) \notin IUD_i$, further, $f(w) \notin IUD_i$, contraction with the precondition;

On the other side, if $w \notin Min_{\preceq_i}(W)$, then there must be a model $M''_{G'}$ before repeated upgrade $\Uparrow \boldsymbol{Ra^{re'}}$ stabilize, so that $M''_{G'}, w \nvDash Ra^{re'}$. So, $\exists i \in N, M''_{G'}, w \nvDash Ra_i^{re'}$, further, we can derive that $f_i(w)$ must be a regret-dominated for i, thus, $f_i(w) \notin IUD_i$, i.e., $f(w) \notin IUD$, which is also contradiction with the precondition.