An Inquisitive Formalization of Interrogative Inquiry

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1 Introduction and motivation

The notion of *interrogative inquiry* refers to the process of *knowledge-seeking by questioning* [5, 6]. As illustrated by the Platonic dialogues [4], conversations constitute one of the main contexts in which the process of interrogative inquiry takes place. Indeed, one might see interrogative inquiry as a *language-game* in which we find ourselves engaged in everyday in order to be brought in specific informational states. Surprisingly, however, little attention has been given so far to the specificities of the process of interrogative inquiry in conversational contexts. From this perspective, a formal modelling of interrogative inquiry would require to attach a particular attention (i) to the formal representation of conversational contexts and (ii) to the logical modelling of questions and answers in conversations.

Recently, significant advances have been made in the field of formal semantics and pragmatics on the representation of questions in discourse. One of the most promising lines of research in this direction is the development of *inquisitive semantics* and *pragmatics* [1, 3]. The main idea of inquisitive semantics is to provide a notion of meaning which incorporates both *informative* and *inquisitive contents*, where the informative content of a sentence refers to its capacity to bring in new information, while its inquisitive content refers to its capacity to raise new issues. The inquisitive notion of meaning allows to build an inquisitive pragmatics which provides a precise modelling of the meaning and the role of questions in conversational information flow. Thus, inquisitive semantics and pragmatics appears as a very suitable framework for investigating the language-game of interrogative inquiry.

The aim of this paper is precisely to develop a formalization of interrogative inquiry based on the inquisitive framework. Indeed, besides its adequacy to represent questions in a conversational context, several features of inquisitive semantics and pragmatics motivate such a project. More specifically, the inquisitive framework offers: (i) a sophisticated modelling of questions which allows to represent *embedded questions*, such as conditional and alternative questions; (ii) a semantic categorization of *questions* and *assertions*; (iii) a precise notion of *answerhood*; (iv) an account of *complete* and *partial answers*. In this paper, we will see that each one of these features turns out to be directly relevant to a formal investigation of interrogative inquiry.

The paper is organized as follows. In section 2, we provide the elements of inquisitive semantics necessary to the logical modelling of questions and answers in the inquisitive framework. In section 3, we discuss and define the notion of *interrogative rule* which aims to characterize completely the question-answer steps that one can make in an interrogative inquiry. In section 4, we put the interrogative rule into a temporal perspective by introducing the notion of *interrogative protocol* which aims to govern interrogative inquiry as a temporal process. This allows us to define the two key notions of *interrogative inquiry* and *interrogative consequence*. One of the main arguments in favor of the framework thus defined lies in its capacity to allow a precise investigation of the logical aspects of interrogative inquiry. this will be illustrated in section 5 where we will relate the logical notion of interrogative consequence to the ones of distributed information and yes-no question.

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2 Modelling questions and answers in inquisitive semantics

In this section, we present the basic elements of inquisitive semantics and pragmatics, following [1] and [3], necessary for the modelling of questions and answers in conversations. We first consider a propositional language $\mathcal{L}$:

Definition 2.1 (Language $\mathcal{L}$). *Let $\mathcal{P}$ be a finite set of propositional variables. The language $\mathcal{L}$ is given by:*

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \rightarrow \psi, \qquad \textit{where } p \in \mathcal{P}.$$

We then define the notion of *support* which in turn will be used to define the notion of *proposition*. To this end, we need to define the notions of *index* and *state*: an *index* v is a binary valuation $v : \mathcal{P} \rightarrow \{0, 1\}$ for the set of propositional variables $\mathcal{P}$, and a *state* is a non-empty set of indices. We will use the following notation: v as a variable ranging over indices, s, t as variables ranging over states, ω to denote the set of all indices and $\mathcal{S}$ to denote the set of all states. We can now define recursively the notion of *support*:

Definition 2.2 (Support). *Let $s \in \mathcal{S}$. The notion of support is defined recursively as follows:*

1. $s \models p$ *iff for all* $v \in s : v(p) = 1$
2. $s \models \neg\varphi$ *iff for all* $t \subseteq s$ *: not* $t \models \varphi$
3. $s \models \varphi \vee \psi$ *iff* $s \models \varphi$ *or* $s \models \psi$
4. $s \models \varphi \wedge \psi$ *iff* $s \models \varphi$ *and* $s \models \psi$
5. $s \models \varphi \rightarrow \psi$ *iff for all* $t \subseteq s$ *: if* $t \models \varphi$ *then* $t \models \psi$

In the above definition, we read $s \models \varphi$ as *state s supports φ*. We now turn to the inquisitive notion of *proposition* which is defined via the notions of *support* and *possibility*:

Definition 2.3 (Possibility, proposition and truth set). *Let $\varphi \in \mathcal{L}$ and $s \in \mathcal{S}$. A possibility for φ in s is a maximal substate of s supporting φ. The proposition expressed by φ in s, denoted by $s[\varphi]$, is the set of possibilities for φ in s. The truth set of φ in s, denoted by $s|\varphi|$, is the set of indices in s where φ is classically true.*

In inquisitive pragmatics, the intended interpretation of the state s to which we relativize our definitions is to represent the *common ground* of the conversation. Thus, the intuitive idea behind inquisitive semantics is to conceive propositions as *proposals* to change the common ground, where the different *possibilities* constitutive of a proposition precisely encode the proposed ways to do so. We can then say that a proposition is *inquisitive* when it consists of more than two possibilities, and *informative* when the union of its possibilities excludes some indices of the common ground. This in turn allows us to define the semantic categories of *questions* and *assertions*:

Definition 2.4 (Informativeness and inquisitiveness). *Let $\varphi \in \mathcal{L}$ and $s \in \mathcal{S}$. We say that: (i) φ is inquisitive in s iff $s[\varphi]$ contains at least two possibilities, (ii) φ is informative in s iff $s[\varphi]$ contains at least one possibility and $\bigcup s[\varphi] \subset s$.*

Definition 2.5 (Question and assertion). *Let $\varphi \in \mathcal{L}$ and $s \in \mathcal{S}$. We say that: (i) φ is a question in s iff φ is inquisitive and not informative in s, (ii) φ is an assertion in s iff φ is not inquisitive and informative in s.*

We now have all the ingredients to define the notion of *answerhood*:

Definition 2.6 (Answerhood). *Let $\varphi, \psi \in \mathcal{L}$ and $s \in \mathcal{S}$ such that ψ is a question and φ an assertion in s. We say that φ is an answer to ψ in s iff (i) $s|\varphi|$ coincides with the union of a set of possibilities for ψ in s and (ii) φ is informative in s.*

The inquisitive framework offers us a powerful theory to represent questions and answers. In the following section, we show how this framework can be used to define the notion of interrogative rule, which will characterize completely the question-answer steps that one can make in an interrogative inquiry.

3 Interrogative rule

The notion of *interrogative rule* should be thought of in analogy with the notion of *inference rule*: where the inference rules are governing the logical inferences that one is allowed to draw, the interrogative rule is governing the admissible question-answer steps that one is allowed to make. In this paper, the aim of the interrogative rule is to characterize completely the question-answer steps that one can make in an interrogative inquiry. For this purpose, we will 'split' the notion of interrogative rule into two components: (i) a pragmatic rule for *answering:* which governs the production of answers to questions given the informational state of the answerer and the common ground of the conversation, (ii) a pragmatic rule for *updating:* which governs the way the conversation, i.e., the common ground and the informational states of the participants, is updated after the reception of an answer to a question. Before defining formally these notions, we first stipulate the basic rules for the language-game of interrogative inquiry, and we capture formally the idea of conversational context by defining the notion of *conversational state.*

Conversational state. In order to make precise the bases of our framework, we first need to stipulate some basic rules of the language-game of interrogative inquiry. To this end, we will set the following hypotheses on the type of conversation which serves as the setting of our investigation of interrogative inquiry: (i) we designate one of the participants as the *inquirer* and the other participants as the *oracles*; (ii) each interrogative step takes the form of a question asked by the inquirer and (eventually) answered by one of the oracles or by the inquirer himself; (iii) each question asked by the inquirer is directed towards a particular conversational participant.

In order to formalize the general informational and conversational context which corresponds to a given stage of a conversation of this type, we introduce the notion of *conversational state*:

Definition 3.1 (Conversational state). *A conversational state C is defined as a tuple $C = (\sigma, \tau_I, \tau_{O_1}, \dots, \tau_{O_n})$ where: σ denotes the common ground of the conversation, τ_I denotes the informational state of the inquirer, and $\tau_{O_1}, \dots, \tau_{O_n}$ denote the informational states of the oracles, and such that: (1) $\tau_I, \tau_{O_1}, \dots, \tau_{O_n} \subseteq \sigma$ and (2) $\left(\bigcap_{1\leq i\leq n} \tau_{O_i}\right) \cap \tau_I \neq \emptyset$. The set of all conversational states is denoted by $\mathcal{C}$, the set of all conversational states with n oracles is denoted by $\mathcal{C}^n$.*

In the above definition, clause (1) says that the informational states of the participants are contained in the common ground of the conversation, clause (2) says that the participants of the conversation have at least one index in common in their respective informational states, which in particular means that they all have the index corresponding to the actual world in their informational states.

Pragmatic rule(s) for answering. By a *pragmatic rule for answering*, we mean a rule that governs the production of answers to questions. In section 2, we provided a general notion of *answerhood* which says when a proposition φ should be considered as an answer to a question ψ in a given state s. However, if we are interested in the production of answers by a given conversational participant, we need to relativize this definition to the informational state of the answerer. To this end, we introduce the notion of *answer* which says when a proposition φ is an answer to a question ψ for an answerer with informational state τ in a conversational state with common ground σ. This is done by requiring the answer to a question to be non-eliminative in the informational state of the answerer:

Definition 3.2 (Answer). *Let $\varphi, \psi \in \mathcal{L}$ and $\sigma, \tau \in \mathcal{S}$ such that $\tau \subseteq \sigma$, ψ is a question and φ an assertion in σ. We say that φ is an answer to ψ for τ in σ iff (i) φ is an answer to ψ in σ and (ii) $\tau|\varphi| = \tau$. We denote by $\mathsf{Answers}(\psi, \tau, \sigma)$ the set of all $\varphi \in \mathcal{L}$ such that φ is an answer to ψ for τ in σ.*

From the notion of answer, we can now define the notion of *answering rule* as a partial function which, for every triple (ψ, τ, σ) such that ψ is a question in σ, picks a formula $\varphi \in \mathcal{L}$ such that φ is an answer to ψ for τ in σ. Formally, this leads to the following definition:

Definition 3.3 (Answering rule)**.** *An answering rule is a partial function*

$$\begin{array}{rccc} A & : & \psi, \tau, \sigma & \longmapsto & A(\psi, \tau, \sigma) \\ & & \mathcal{L} \times \mathcal{S} \times \mathcal{S} & \longrightarrow & \mathcal{L} \end{array}$$

where $A(\psi, \tau, \sigma)$ is defined for all (ψ, τ, σ) such that ψ is a question in σ by

$$A(\psi, \tau, \sigma) = \begin{cases} \varphi \in \mathsf{Answers}(\psi, \tau, \sigma) & \textit{if } \mathsf{Answers}(\psi, \tau, \sigma) \neq \emptyset, \\ \top & \textit{otherwise.} \end{cases}$$

Pragmatic rule for updating. In the previous paragraph, we defined a pragmatic rule for answering which governs the production of answers to questions. We then need to state how a produced answer *modifies* the informational states of the conversational participants along with the common ground of the conversation, i.e., how it modifies the current *conversational state*. To this end, we define an *updating rule* which maps a conversational state and a sentence to the conversational state updated after the utterance of this sentence:

Definition 3.4 (Updating rule)**.** *The updating rule is a partial function*

$$\begin{array}{rccc} U & : & C, \varphi & \longmapsto & C|\varphi \\ & & \mathcal{C} \times \mathcal{L} & \longrightarrow & \mathcal{C} \end{array}$$

where $C|\varphi$ is defined for all (C, φ) such that $s|\varphi| \neq \emptyset$, with $s := \left(\bigcap_{1 \leq i \leq n} \tau_{O_i}\right) \cap \tau_I$ and $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$, by

$$C|\varphi = (\sigma|\varphi, \tau_I|\varphi, \tau_{O_1}|\varphi, \ldots, \tau_{O_n}|\varphi), \textit{ with } t|\varphi = t|\varphi| \textit{ for } t \in \mathcal{S}.$$

In this definition, we adopt the most straightforward solution for updating conversational states which consists in ruling out the indices in the informational states of the conversational participants, along with the indices in the common ground of the conversation, which are incompatible with the uttered sentence. Notice that the updating rule preserves conversational states, which means that, if (C, φ) is such that $s|\varphi| \neq \emptyset$, where $s := \left(\bigcap_{1 \leq i \leq n} \tau_{O_i}\right) \cap \tau_I$, then $U(C, \varphi)$ is a conversational state according to definition 3.1.

Interrogative rule. Having defined the pragmatic rules for answering and updating, we can now straightforwardly define from these two components the notion of *interrogative rule*:

Definition 3.5 (Interrogative rule)**.** *Let $n \in \mathbb{N}$ and let A be an answering rule. The interrogative rule associated to A and n is a partial function*[1]

$$\begin{array}{rccc} I_n & : & C, \psi, i & \longmapsto & C|_i^? \psi \\ & & \mathcal{C}^n \times \mathcal{L} \times [\![0, n]\!] & \longrightarrow & \mathcal{C} \end{array}$$

where $C|_i^?\psi$ is defined for all (C, ψ, i) such that ψ is a question in σ by $C|_i^?\psi = C|A(\psi, \tau_i, \sigma) = U(C, A(\psi, \tau_i, \sigma))$.

This definition characterizes completely the question-answer steps that an inquirer can make in an interrogative inquiry. However, interrogative inquiries are temporal processes composed of *sequences* of question-answer steps. The aim of the next section will precisely be to put the interrogative rule into a temporal perspective by introducing the notion of *interrogative protocol*.

[1] In a conversational state $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n})$, we will take as a convention to refer to the informational state of the oracle τ_{O_i} as the informational state indexed by i, and to refer to the informational state of the inquirer τ_I as the informational state indexed by 0.

4 Interrogative protocol, interrogative inquiry and interrogative consequence

The notion of interrogative rule defined in the previous section describes completely the epistemic effect of asking a question in a given conversational context. However, an interrogative inquiry is a *sequence* of question-answer steps, and is thereby a *temporal process.* We shall then account for this temporal dimension, and put the interrogative rule into a temporal perspective. To this end, our approach will consist in representing, in a temporal framework, the two main aspects of the language-game of interrogative inquiry that we defined so far: (i) the basic rules of the language-game of interrogative inquiry and (ii) the pragmatic rules for answering and updating. In logic and computer science, such temporal processes are represented using the notion of *protocol*, which refers to a set of rules governing a *temporal process.* In our context, we are interested in defining *interrogative protocols* describing all the possible paths that an interrogative inquiry can take. We will then be able to formally define the notion of *interrogative inquiry* as finite paths within such protocols, and the notion of *interrogative consequence* as the information that can be reached through this process. In this section, we will develop this approach by providing formal definitions for the notions of *interrogative protocol*, *interrogative inquiry* and *interrogative consequence.*

Interrogative protocol. The notion of *protocol* has previously been used in logical contexts to capture the dynamics of informational flow in conversations such as in [9] and [10]. It has even been used to represent questioning procedures in the framework of dynamic epistemic logic of questions [11]. In this paper, we are interested in defining *interrogative protocols* governing the language-game of interrogative inquiry. Our interrogative protocols are built from two parameters: (i) a conversational state constituting the starting point of the conversation, and (ii) an interrogative rule which integrates the adopted pragmatic rules for answering and updating. Besides, our notion of interrogative protocol encodes the basic rules of the language-game of interrogative inquiry by stipulating that the only admissible moves are questions addressed by the inquirer to the conversational participants. This leads to the following formal definition:

Definition 4.1 (Interrogative protocol)**.** *Let $n \in \mathbb{N}$, $C \in \mathcal{C}^n$ and I_n be an interrogative rule. The interrogative protocol $P_?(C, I_n)$ based on C and I_n is defined as a tree built as follows:*

Root: *the root of the tree is C,*

Expanding rule: *if $C' = (\sigma, \tau_I, \tau_{O_1}, \dots, \tau_{O_n})$ is a node of the tree, then for each formula φ such that φ is a question in σ and for each $i \in [\![0, n]\!]$, C' has a successor $C'|^?_i\varphi = I_n(C', \varphi, i)$.*

Interrogative inquiry. From the definition of interrogative protocols, we can then reach a formal definition of the notion of *interrogative inquiry.* To this end, we first notice that, for each node C of an interrogative protocol, each edge starting from C is identified by a *directed question*, which constitutes the *label* of this edge from C. This means that, in an interrogative protocol, any finite branch from the root can be identified by a finite sequence of labels, i.e., by a finite sequence of directed questions. This gives us our formal definition of the notion of *interrogative inquiry*:

Definition 4.2 (Interrogative inquiry)**.** *Let $P_?(C, I_n)$ be an interrogative protocol. An interrogative inquiry in $P_?(C, I_n)$ is a finite sequence $\langle(\varphi_1, i_1), \dots, (\varphi_k, i_k)\rangle_k$ of elements in $\mathcal{L} \times [\![0, n]\!]$ such that $\langle(\varphi_1, i_1), \dots, (\varphi_k, i_k)\rangle_k$ corresponds to the labels of a finite branch in $P_?(C, I_n)$ from the root C.*

This definition fits our intuitive representation of interrogative inquiries as sequences of questions addressed to specific conversational participants, i.e., as sequences of directed questions.

Moreover, this definition also fits the idea that an interrogative inquiry takes place in a particular temporal process governs by certain rules. In our case, these rules are represented into an interrogative protocol which encodes (i) the particular type of conversation within which the interrogative inquiry is taking place, (ii) the pragmatic rule for answering governing the production of answers and (iii) the pragmatic rule for updating governing the modifications of conversational states.

Interrogative consequence. In the framework of interrogative logic [7], Hintikka and colleagues introduced the notion of *interrogative derivability*: C is interrogatively derivable from the initial set of premisses T in the model M if there exists a sequence of interrogative and deductive steps, made according to the rules of interrogative logic, leading to the conclusion C. In our framework, we will introduce an analogous notion of *interrogative consequence*: φ is an *interrogative consequence* in the interrogative protocol $P_?(C, I_n)$ if there exists an interrogative inquiry in $P_?(C, I_n)$ leading to a conversational state in which φ has been *established* in the common ground. However, in the inquisitive framework, there are two different possible ways to think of the term *established* here, corresponding to the *classical* and the *inquisitive views* on meaning:

- *Classically*, we consider that a proposition φ has been established in the common ground σ when φ is true in all the indices of σ, i.e., $\sigma|\varphi| = \sigma$,
- *Inquisitively*, we consider that a proposition φ has been established in the common ground σ when φ is composed of only one possibility covering σ, i.e., $\sigma[\varphi] = \{\sigma\}$.

In the classical view, we consider that a state encodes information φ as soon as φ is classically true in all the indices of the considered state, which represents then the current range of epistemic possibilities. However, due to the capacity of inquisitive semantics to encode both informative and inquisitive contents, the classical view is not enough in our case: φ can be classically true in all the indices composing the common ground while still being *inquisitive*, i.e., while still raising some issues. Thus, this observation speaks for a stronger[2] notion in which we will consider that a formula φ has been established in the common ground if not only φ is classically true in all the indices composing the common ground, i.e., φ is not informative in the common ground, but also φ does not raise any more issues. This precisely amounts to say that φ has been *settled* in the common ground. Consequently, we will adopt the following definition of the notion of *interrogative consequence*:

Definition 4.3 (Interrogative consequence)**.** *Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$. We say that φ is an interrogative consequence in $P_?(C, I_n)$ iff there exists an interrogative inquiry in $P_?(C, I_n)$ leading to a conversational state C' in which φ is settled, i.e., $\sigma'[\varphi] = \{\sigma'\}$.*

We now investigate further logical properties of the notion of interrogative consequence.

5 Logical aspects

In this section, we investigate two issues related to the logical notion of interrogative consequence. The first one is concerned with a characterization of the information reachable by the process of interrogative inquiry in terms of the information possessed by the conversational participants, and shows how the notions of interrogative consequence and distributed information are formally related. The second one is concerned with the information reachable by interrogative inquiries exclusively composed of yes-no questions, and shows that any interrogative consequence is reachable by only asking yes-no questions.

[2] This stronger property is a formal feature of the framework: if $\sigma \in \mathcal{S}$, $\varphi \in \mathcal{L}$ and $\sigma[\varphi] = \{\sigma\}$, then $\sigma|\varphi| = \sigma$.

5.1 Interrogative consequence and distributed information

In the context of epistemic logic, the notion of distributed information intuitively refers to the information that a group of epistemic agents would have if they would put all the information they individually have together. This notion is generally semantically defined as follows [2, 8]: φ is distributed information among a group of agents G if and only if φ is true in all the worlds that every agent in G considers epistemically possible. In our framework, we are interested in defining a notion of distributed information among the participants of a conversation. To this end, we propose the following definitions:

Definition 5.1 (Distributed information). *Let $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n}) \in \mathcal{C}^n$ and $\varphi \in \mathcal{L}$. (i) We define the distributed information state $D(C)$ of C as $D(C) := \bigcap_{1 \leq i \leq n} \tau_{O_i} \cap \tau_I$. (ii) We say that φ is distributed information in C iff φ is settled in $D(C)$, i.e., $D(C)[\varphi] = \{D(C)\}$. (iii) We define the saturated conversational state C_D of C as $C_D := (D(C), \ldots, D(C))$.*

Intuitively, we would expect that whenever φ is distributed information among the participants of a conversation, φ can be reached by the inquirer through the process of interrogative inquiry. In other words, we would expect that the notions of distributed information and interrogative consequence *coincide*. This intuitive relation between distributed information and interrogative consequence can formally be established in our framework:

Theorem 1 (Interrogative Consequence and Distributed Information). *Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$.*

φ is an interrogative consequence in $P_?(C, I_n)$ iff φ is distributed information in C.

Proof. See appendix A. □

5.2 Interrogative consequence and yes-no questions

In the framework of interrogative logic [7], Hintikka and colleagues proved the so-called *yes-no theorem.* This theorem says that, whenever a conclusion C can be established through an interrogative inquiry, C can be established by only asking *yes-no questions.* In our framework, we propose the following definition of *yes-no questions*:

Definition 5.2 (Yes-no question). *Let $\varphi \in \mathcal{L}$. We define the yes-no question associated to φ by*

$$!\varphi \vee \neg !\varphi.$$

The idea of this definition is, for a given formula $\varphi \in \mathcal{L}$, to first transform φ in an assertion $!\varphi$ using the operator $!$,[3] and then construct the yes-no question $!\varphi \vee \neg !\varphi$. Given this definition, we can now formally show that the yes-no theorem holds in our framework:

Theorem 2 (Yes-no theorem). *Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$. If φ is an interrogative consequence in $P_?(C, I_n)$, then there exists an interrogative inquiry composed exclusively of yes-no questions which settles φ, i.e., which leads to a conversational state C' such that $\sigma'[\varphi] = \{\sigma'\}$.*

Proof. See appendix B. □

What the yes-no theorem says is that, whenever φ is an interrogative consequence in an interrogative protocol $P_?(C, I_n)$, φ can be reached in $P_?(C, I_n)$ through an interrogative inquiry composed exclusively of yes-no questions.

[3] The operator ! is defined as a shortcut for double negation, so that $!\varphi$ is a shortcut for $\neg\neg\varphi$. This operator has for effect to transform any sentences into an assertion. See [3, p. 4] for more details on this operator.

6 Conclusion

In this paper, we have developed a formal framework for investigating the process of interrogative inquiry in conversational contexts. Our approach was built on recent developments on the modelling of questions and answers in conversations, and took as its foundations the framework of inquisitive semantics and pragmatics. We have proposed a way to define the key notions of *interrogative rule*, *interrogative protocol*, *interrogative inquiry* and *interrogative consequence*, and we hope to have convinced the reader that the inquisitive framework was very suitable for doing so. We have then shown how our formal framework enables subtle logical investigations of the process of interrogative inquiry. This paper is a very first step towards connecting the inquisitive framework with the study of interrogative inquiry.

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Appendix

A Proof of Theorem 1

Theorem 1 (Interrogative Consequence and Distributed Information)**.** *Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$.*

φ is an interrogative consequence in $P_?(C, I_n)$ iff φ is distributed information in C.

Proof. Let $P_?(C, I_n)$ be an interrogative protocol, with $C = (\sigma, \tau_I, \tau_{O_1}, \ldots, \tau_{O_n}) \in \mathcal{C}^n$, and let $\varphi \in \mathcal{L}$. Assume that φ is an interrogative consequence in $P_?(C, I_n)$. By definition, this means that there exists an interrogative inquiry $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ in $P_?(C, I_n)$ leading to a node $C' = (\sigma', \tau'_I, \tau'_{O_1}, \ldots, \tau'_{O_n})$ such that $\sigma'[\varphi] = \{\sigma'\}$. We will show that $D(C) \subseteq \sigma'$.

Let $v \in D(C)$. Suppose towards a contradiction that $v \notin \sigma'$. Since $v \in \sigma$, this would mean that the announcement of the answer χ_p to φ_p for some $p \in [\![1, k]\!]$ has led to the elimination of v. However, since $v \in D(C)$, this means that v is a member of $\tau_I, \tau_{O_1}, \ldots, \tau_{O_n}$, i.e., a member of the informational state of each conversational participant. Thus, the announcement of χ_p must have been eliminative in the informational state τ_{i_p} of the participant with index i_p, which is not possible due to the definition of the notion of answer.[4] By contradiction, we get that $v \in \sigma$ and finally that $D(C) \subseteq \sigma'$. Then, it follows from $D(C) \subseteq \sigma'$ and $\sigma'[\varphi] = \{\sigma'\}$ that $D_C[\varphi] = \{\varphi\}$, i.e., that φ is distributed information in C.

Now assume that φ is distributed information in C. By definition, this means that $D(C)[\varphi] = \{D(C)\}$. We will show that there exists an interrogative inquiry in $P_?(C, I_n)$ which leads to the saturated conversational state C_D of C. Consider the following interrogative inquiry:

$$\langle(\chi_{\tau_I}?, 0), (\chi_{\tau_{O_1}}?, 1), \ldots, (\chi_{\tau_{O_n}}?, n)\rangle_{n+1}.$$

We claim that this interrogative inquiry leads to to the saturated conversational state C_D. To see this, consider the sequence $\langle C_1, \ldots, C_{n+1}\rangle_{n+1}$ of conversational states associated to $\langle(\chi_{\tau_I}?, 0), (\chi_{\tau_{O_1}}?, 1), \ldots, (\chi_{\tau_{O_n}}?, n)\rangle_{n+1}$. We then get that:

$$\begin{array}{rcl}
\sigma_1 & = & \sigma|\chi_{\tau_I} = \tau_I, \\
\sigma_2 & = & \sigma|\chi_{\tau_I}|\chi_{\tau_{O_1}} = \tau_I \cap \tau_{O_1}, \\
& \vdots & \\
\sigma_{n+1} & = & \sigma|\chi_{\tau_I}|\chi_{\tau_{O_1}}|\ldots|\chi_{\tau_{O_n}} = \tau_I \cap \bigcap\limits_{1 \leq i \leq n} \tau_{O_i},
\end{array}$$

where σ_q denotes the common ground of the conversational state C_q. Thus, we get that $C_{n+1} = C_D$, which means that the interrogative inquiry $\langle(\chi_{\tau_I}?, 0), (\chi_{\tau_{O_1}}?, 1), \ldots, (\chi_{\tau_{O_n}}?, n)\rangle_{n+1}$ leads to the saturated conversational state C_D of C. From that and the initial assumption that φ is distributed information in C, i.e., $D(C)[\varphi] = \{D(C)\}$, we conclude that φ is an interrogative consequence in $P_?(C, I_n)$. □

B Proof of Theorem 2

Theorem 2 (Yes-no theorem)**.** *Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$. If φ is an interrogative consequence in $P_?(C, I_n)$, then there exists an interrogative inquiry composed exclusively of yes-no questions which settles φ, i.e., which leads to a conversational state C' such that $\sigma'[\varphi] = \{\sigma'\}$.*

[4] In other words, this would mean that the answer χ_p produced by the participant indexed i_p does not respect the 'informative sincerity' clause of the sincerity maxim.

Proof. Let $P_?(C, I_n)$ be an interrogative protocol and $\varphi \in \mathcal{L}$. Assume that φ is an interrogative consequence in $P_?(C, I_n)$. By definition, this means that there exists an interrogative inquiry $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ in $P_?(C, I_n)$ leading to a conversational state C' such that $\sigma'[\varphi] = \{\sigma'\}$. Now, let $\chi_1, \ldots, \chi_k$ be the answers respectively obtained as responses to the directed questions of the sequence $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$. Then, we claim that $\langle(\chi_1?, i_1), \ldots, (\chi_k?, i_k)\rangle_k$ is an interrogative inquiry in $P_?(C, I_n)$ leading to the same node C'.[5] To see this, notice that for any $q \in [\![1, k]\!]$, if χ_q is the answer to φ_q for i_q in C_q, where C_q denotes the node corresponding to the end-point of the branch $\langle(\varphi_1, i_1), \ldots, (\varphi_q, i_q)\rangle_q$, the answer to $\chi_q?$ for i_q in C_q is necessarily χ_q. The sequence of conversational states $\langle C_1, \ldots, C_k\rangle_k$ resulting from $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ in $P_?(C, I_n)$ is then identical to the sequence of conversational states associated to $\langle(\chi_1?, i_1), \ldots, (\chi_k?, i_k)\rangle_k$. Consequently, the two interrogative inquiries $\langle(\varphi_1, i_1), \ldots, (\varphi_k, i_k)\rangle_k$ and $\langle(\chi_1?, i_1), \ldots, (\chi_k?, i_k)\rangle_k$ lead to the same conversational state C', i.e., to a conversational state in which φ is settled. Hence, we have found an interrogative inquiry composed exclusively of yes-no questions which settles φ, namely $\langle(\chi_1?, i_1), \ldots, (\chi_k?, i_k)\rangle_k$. $\square$

[5] Since $\chi_1, \ldots, \chi_k$ are assertions, as answers to some questions, we write $\chi_i?$ instead of $(!\chi_i)?$ for the yes-no question associated to χ_i.