Short Sight, Priority Graph and Preference Change in Games

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Abstract. Motivated by real game scenarios, we propose a generalized prioritized game model in which players' preference and reasons can be studied altogether. We show that the incomparable relation between two histories in game is natural, hence calls for a new modeling. We generalize the main research results of [2] to the situations which allow for incomparable priorities. We look at two kinds of dynamics changes in games with short sight, changes in priority graph and changes in sight. We show that changes of priority in games with short sight result in changes of preference in its canonical representation, and changes of short sight may also lead to preference change in the corresponding games with awareness.

1 Introduction

In classical game theory, there are two assumptions of common knowledge and rationality. Namely, the specification of the game are known to all players. And all players are rational in the sense that they prefer strategies that will maximize their individual expected utilities. These assumptions have been noted to be too strong and unrealistic and several attempts have been made to achieve a closer match with reality. Halpern et al. study the issue of unawareness [1, 6], in which players may have no access to the whole game tree when they make decisions because of their ignorance of other player's strategies. More recently, Grossi and Turrini put up the idea of games with short sight in [2] where players might neither see a part of the terminal nodes of a game tree nor even see any such nodes. This has advanced Halpern's stance. However, we still see substantial room for improving the previous proposals.

As a motivating example, we first tell a story.

Once there was a girl named Alice, who was in a chess family. Alice had a lot of wooers because she was really beautiful, elegant and smart. Her father wanted her to marry a chess expert, so they hold a chess tournament, the champion of which could marry Alice. The rule was: each contestant had to play a round separately with three elders in Alice's family. He had to checkmate the first elder then checkmate the second elder in ten steps and finally ensure that, throughout the course of playing with the third elder, the contestant hold more chessmen than his opponent. The requirement was so harsh that almost all contestants were eliminated. Eventually, a young man Dave successfully passed the three rounds. Alice's father, who was a chess master, was so excited for finding his match that he nearly forgot Alice's marriage and asked Dave to play a round with him. Dave, who could hardly wait to marry Alice, agreed to get another one but this time he only expected they can finish it quickly without caring about the outcome. Finally, Alice and Dave got married.

There are various criteria that players will consider when making decisions. Take the scenarios in the story for example, the criteria may involve (1) when playing with the first elder, Dave prefers the histories where he can checkmate his opponent (called *checkmated*, denoted as cm); (2)Dave would not be satisfied with just winning, but prefers to checkmate the second elder in fewer steps(called *step advantage*, denoted as sp); (3) Dave prefers the histories that can make him hold as many chessmen as possible in the third round (called *material advantage*, denoted as ma); (4) In order to marry Alice earlier, Dave does not care about the result when playing with Alice's father, he just prefers the histories where the game will finish more quickly (called *time advantage*, denoted as ta).

As analyzed above, a player's preference is formed from various criteria, more importantly, the priority among them is different. A simplest case would be that they are linearly ordered and the player can compare everything. But in reality we often cannot make comparisons. For instance, ta and ma might be incomparable to us. We are interested in studying preference and priorities in the same model.

To move further, we can also see from the story that ordered priorities allow for dynamic changes. Some properties may become more important to agents' preferences, and some may become less important, or totally irrelevant.

To account for the role of priorities in forming player's preference and possible changes, in this paper we will use ideas from [5] and introduce a representation of priority-based preference in games: instead of having player's usual preference relation, we introduce player's priority on properties of histories, obtaining prioritized games (in Section 2). Then we study the dynamics of priority graph and the changes of graph-preference(in Section 3). The situation of incomparable priorities is captured by our generalization from the linearly ordered priority sequence in [2] to priority graph. Accordingly, we obtain the corresponding generalized results on game with short sight. Further, we study the dynamics in the representation theorem. More specifically, we show that changes of priority in games with short sight result in changes of preference in its canonical representation and changes of short sight may also lead to preference dynamics in the corresponding games with awareness (in Section 4). Conclusions and Future work are showed in Section 5. Because of the limitation of length, most proofs for propositions, theorems and facts are added in the Appendix.

2 Priority-based games

In this section, we first define priority graph and the graph-induced preference. Then we will introduce games that are endowed with priority graph, which is called *prioritized games*. To do that, we introduce extensive game now.

Definition 1. *(Extensive game) An extensive game is a tuple* $\Gamma = (N, H, t, \Sigma_i, O, \succeq_i)$ *where:*

- N *is a non-empty set consisting of the players of the game.*
- H *is a non-empty set of sequences, called histories, that satisfies the following three properties:*

◦ *The empty sequence* $\emptyset$ *is a member of* H*;*

◦ *If* $(a^k)_{k=1,\ldots,K} \in H$ *and* $L < K$ *then* $a^k_{k=1,\ldots,L} \in H$*;*

◦ *If an infinite sequence* $(a^k)_{k=1,\ldots}$ *is such that* $(a^k)_{k=1,\ldots,L} \in H$ *for every positive integer* L *then*$(a^k)_{k=1,\ldots} \in H$

A history $(a^k)_{k=1,\ldots,K} \in H$ *is called terminal history or run if it is infinite or if there is no* a^{K+1} *such that* $(a^k)_{k=1,\ldots,K+1} \in H$*. The set of terminal histories is denoted* Z*.*

If h *is a prefix of* h' *we write* $h \lhd h'$*.*

Each component of a history is called an action. The set of all actions is denoted A*.* $A_h = \{a|(h,a) \in H\}$*, presenting the set of actions following the history* h*.*

- $t : H \setminus Z \to N$ *is a function, called turn function, assigning a member of* N *to each non-terminal history.* ($t(h)$ *is the player who takes an action after the history* h*, i.e. player* i *moves at history* h *whenever* $t(h) = i$*);*

Let $H_i = \{h|t(h) = i\}$ *be the set of all histories after which player* i *moves.*

- Σ_i *is a non-empty set of strategies. A strategy of player* i *is function* $\sigma_i : \{h \in H \backslash Z|\ t(h) = i\} \to A_h$ (*assigns an action in* A_h *to each non-terminal history for which* $t(h) = i$*);*
- O *is the outcome function. For each strategy profile* $\Sigma = (\sigma_i)_{i \in N}$*, the outcome* $O(\Sigma)$ *of* Σ *is the terminal history that results when each player* i *follow the precepts of* σ_i*. That is,* $O(\Sigma)$ *is the history* $(a^1, \ldots, a^K) \in Z$ *such that* $0 \leq k < K$*, we have* $\sigma_{t(a^1,\ldots,a^k)} = a^{k+1}$*. Formally,* $O : \Pi_{i \in N}\Sigma_i \to Z$*.*
- $\succeq_i \subseteq Z^2$ *is a total preorder over* Z*, for each player* i(*the preference relation for each player* i).

We also call the tuple (N, H, t, Σ_i, O) *extensive game form* according to [2], and use the notation $\mathcal{G}$ to denote it for later use.

Remark 1. We will illustrate it soon, the meaning of the notation $\succeq_i$ here is different from that in the previous literatures.

In [2], properties are taken to be sets of game positions, i.e., sets of histories, and the priority order on properties is linear. We generalize their model by priority graph which is defined as follows.

Definition 2. *(Priority Graph) Let $\mathcal{G}$=(N, H, t, Σ_i, O) be an extensive game form. A priority graph,or P-graph, for $\mathcal{G}$ is a tuple g =$(\mathcal{H}, \succ)$ where:*

• *$\mathcal{H} \subseteq \wp(H)$ and $\mathcal{H}$ is finite, i.e., the set of properties $\mathcal{H}$ is a finite set of sets of histories. Elements of $\mathcal{H}$ are denoted $\mathbf{H}, \mathbf{H}' \ldots$*

• *$\succ \subseteq \mathcal{H}^2$ is a strict partial order on properties in $\mathcal{H}$. To say that $\mathbf{H}$ is preferred to $\mathbf{H}'$, for $\mathbf{H}, \mathbf{H}' \in \mathcal{H}$, we write $\mathbf{H} \succ \mathbf{H}'$.*

Priority graphs express the priority order of a finite set of relevant criteria using which the players can assess game positions. In our view, they represent the reason for players' preference over histories.

Given a P-graph, a preference over histories can be derived in a natural way:

Definition 3. *(Preferences). Let $\mathcal{G} = (N, H, t, \Sigma_i, O)$ be an extensive game form and g =$(\mathcal{H}, \succ)$ a P-graph for $\mathcal{G}$. The preference relation $\succsim^g \subseteq H^2$ over the set of histories of $\mathcal{G}$ induced by g is defined as:*

$$\succsim^g = >^g \cup \backsim^g \cup \bowtie^g$$

the notations in this concept are defined as follows:

$$\begin{aligned} h \gtrsim^g h' \Longleftrightarrow \forall \mathbf{H} \in \mathcal{H} : [h' \in \mathbf{H} \Longrightarrow h \in \mathbf{H} \\ \vee \quad \exists \mathbf{H}' \in \mathcal{H} : [\mathbf{H}' \succ \mathbf{H} \wedge h \in \mathbf{H}' \wedge h' \notin \mathbf{H}']] \end{aligned}$$

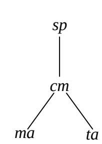
$$h >^g h' \Leftrightarrow h \gtrsim^g h' \wedge \neg(h' \gtrsim^g h)$$

$$h \backsim^g h' \Leftrightarrow h \gtrsim^g h' \wedge h' \gtrsim^g h$$

$$h \bowtie^g h' \Longleftrightarrow \neg(h \gtrsim^g h' \vee h' \gtrsim^g h)$$

Therefore, there are four kinds of preference relations over histories induced by the priority graph.

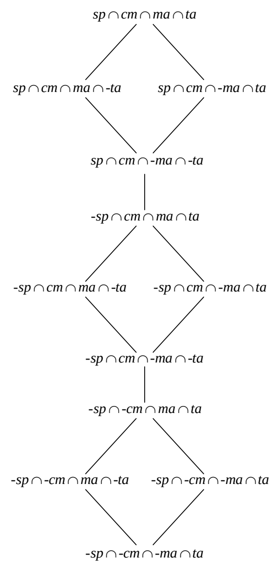
Example 1. Suppose the priority graph g is:



g

The preference relation P over histories derived from g can be represented by the figure below(Actually, this is not the complete preference relation. Those nodes containing $sp \cap -cm$ are omitted since they are impossible considering the meaning of sp and cm).

We can see that players prefer the positions where he can checkmate his opponents with step advantage, material advantage and time advantage above all others. Obviously, graph-induced preference relation is also a partial order rather than liner one. Thus we call it *preference graph*.



P

Theorem 1. *For two strategy profiles σ and σ' with $O(\sigma) = h$ and $O(\sigma') = h'$, if h and h' are at the same level of the preference graph, then σ and σ' are indistinguishable.*

Fact 1. From the perspective of equilibrium in games, $h \bowtie^g h'$ can be seen as the same as $h \backsim^g h'$.

Remark 2. According to Fact 1, it seems that the definition of $\succsim^g$ can be rewritten as $\succsim^g = >^g \cup \backsim^g$, we will take this method in the rest of this paper, but we need to notice that the meaning of $\backsim^g$ has changed(including the relation of $\bowtie^g$).

Fact 2. Let $\mathcal{G}$ be an extensive game form and $g = (\mathcal{H}, \succ)$ a P-graph for $\mathcal{G}$. The relation $\succsim^g$ has the following properties:

1. It is a total pre-order;
2. $\succsim^g$ contains at most $2^{|\mathcal{H}|}$ equivalence classes.

Introducing priority graph results in an extension of the standard concept of games. In the following definition, we provide the higher level representation of games, in which preference orders in extensive games are substituted by a family of P-graphs, one for each player:

Definition 4. *(Prioritized games). A prioritized game is a tuple $\mathcal{G}^g = (N, H, t, \Sigma_i, O, g_i)$, where g_i be a P-graph for $\mathcal{G}^g$.*

The advantage to have such models is that we can derive preference relation from priorities whenever it is needed. Obviously, each prioritized game $\mathcal{G}^g = (N, H, t, \Sigma_i, O, g_i)$ defines a game in extensive form $\Gamma = (N, H, t, \Sigma_i,\ O, \succeq_i)$, with $\succeq_i = Z^2 \cap \succsim^{g_i}$. So, when attention is restricted to terminal histories, prioritized games yield standard extensive form games.

3 Dynamics of priority graph and preference

As showed in the motivating example, priorities among the criteria for the evaluation of game positions are unlikely to remain the same. To characterize the changes in priority, we now work on the basic update operations on priority graphs.

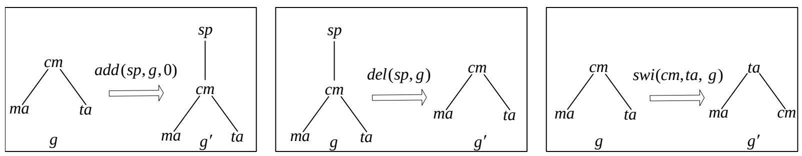
Definition 5. *(Priority graph dynamics) Let $\mathcal{G} = (N, H, t, \Sigma_i, O)$ be an extensive game form and $g = (\mathcal{H}, \succ)$ a P-graph for $\mathcal{G}$. Some natural dynamic operators are listed below, together with examples:*

- add($\mathbf{H}, g, m$): add the property $\mathbf{H}$ to the m_{th} level of the priority graph g
- del($\mathbf{H}, g$): delete the property $\mathbf{H}$ from priority graph g
- swi($\mathbf{H}, \mathbf{H}', g$): switch the positions of the two properties $\mathbf{H}$ and $\mathbf{H}'$ in priority graph g

Example 2. Suppose the priority graph is g at first, but now the player wants to checkmate the opponent in as few steps as possible, then sp should be added in the top of the priority graph.

Example 3. Contrary to the example of adding a node, now assume that the player does not care about how many steps needed to checkmate the opponent, then the node sp should be deleted:

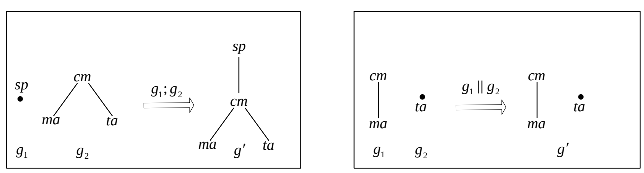
Example 4. Now consider that the player prefer ta most, then the position should be changed:



We can continue to explore more examples and natural operators, but in what follows we would like to follow the modular study in [4] and [3] and consider the two basic operators on P-graphs:

- $g_1; g_2$: the sequential composition, adds the graph g_1 on top of g_2 in the order. Then all nodes in the first come before all those in the second.
- $g_1 \parallel g_2$: the parallel composition, gets the disjoint union of the graphs g_1 and g_2, without any order links between them.

Example 5.



Changes of priority graph would cause the dynamics of graph-induced preference relations. In the following, we explore the correlations between the dynamics of the two levels. Below are some algebraic laws for some of the dynamic operators on priority graphs. We adapt this result from [5]:

Fact 4. The following laws hold for graph-induced preference relations:

(1) $\succsim^{g_1 \| g_2} = \succsim^{g_1} \cap \succsim^{g_2}$

(2) $\succsim^{g_1;g_2} = (\succsim^{g_1} \cap \succsim^{g_2}) \cup \succsim^{g_1 >}$ (notice that for a relation R, $mR^{>}n$ denotes that m is strictly better than n)

As defined before, preference can be derived from P-graph, so changes of priority graph lead to changes in preference. A representation theorem related to this will be showed formally in next section.

4 New Results about Short Sight

On the basis of the two sections above, some generalized results on short sight in games would be discussed comparing with the work in [2].

4.1 Basic notations

First we introduce some basic notations about short sight first proposed in [2].

Definition 6. *(Sight function). Let $\mathcal{G}^g = (N, H, t, \Sigma_i, O, g_i)$ be a prioritized game. A short sight function for $\mathcal{G}^g$ is a function*

$$s : H \backslash Z \to 2^H \backslash \emptyset$$

associating to each non-terminal history h a finite subset of all the available histories at h. That is:

1. $s(h) \in 2^{H|h} \backslash \emptyset$ and $|s(h)| < \omega$, i.e. the sight at h consists of a finite nonempty set of histories extending h

2. $h' \in s(h)$ implies that $h'' \in s(h)$ for every $h'' \lhd h'$, i.e. players' sight is closed under prefixes.

Intuitively, the function associates to any choice point those histories that the player playing at that choice point can see.

Definition 7. *(Games with short sight). A game with short sight is a tuple $S = (\mathcal{G}^g, s)$ where $\mathcal{G}^g$ is a prioritized game and s a sight function for $\mathcal{G}^g$.*

The fact below shows that each game with short sight yields a family of finite extensive games, one for each history $h \in H \backslash Z$:

Fact 3. Let $S = (\mathcal{G}^g, s)$ be a prioritized game with short sight, with $\mathcal{G}^g = (N, H, t, \Sigma_i, O, g_i)$. Let also $h \in H \backslash Z$. Then the tuple $\Gamma\lceil_h$ defined as follows is a finite extensive game:

$\Gamma\lceil_h = ((N\lceil_h, H\lceil_h, t\lceil_h, \Sigma_i\lceil_h, O\lceil_h), \succeq_i \lceil_h)$

where:

- $N\lceil_h = N$;
- $H\lceil_h = s(h)$. The set $Z\lceil_h$ denotes the histories in $H\lceil_h$ of maximal length, i.e., the terminal histories in $H\lceil_h$;
- $t\lceil_h = H\lceil_h \backslash Z\lceil_h \to N$ so that $t\lceil_h(h') = t(h, h')$;
- $\Sigma_i\lceil_h$ is the set of strategies for each player available at h and restricted to $s(h)$. It consists of elements $\sigma_i\lceil_h$ such that $\sigma_i\lceil_h(h') = \sigma_i(h, h')$ for each $h' \in H\lceil_h$ with $t\lceil_h(h') = i$;
- $O\lceil_h$: $\prod_{i \in N} \Sigma_i\lceil_h \to Z\lceil_h$;
- $\succeq_i\lceil_h = \succsim^{g_i} \cap (Z\lceil_h)^2$.

Remark 3. From the last item of the fact above, we can see that although we follow the notation $\succeq_i$ in [2], we endow it with a richer meaning: Since the relation $\succsim^{g_i}$ contains incomparable relation $\bowtie^{g_i}$ and $\succeq_i$ is deduced from $\succsim^{g_i}$, the meaning of $\succeq_i$ here also includes the circumstance that two histories are incomparable.i.e., for two terminal histories $h, h' \in Z\lceil_h$, if $h \bowtie^{g_i} h'$, then we will have $h \succeq_i\lceil_h h'$.

Then, we will discuss games with possibly unaware players and lack of common knowledge[1].

An augmented game ${}^+\Gamma$, which is associated to each extensive game $\Gamma = (N, H, t, \Sigma_i, O, \succeq_i)$, specifies level of awareness of each player at each node of the original game.

Definition 8. *(Augmented game) Let $\Gamma = (\mathcal{G}, \succeq_i)$ be a finite extensive game with $\mathcal{G} = (N, H, t, \Sigma_i, O)$ The augmented game ${}^+\Gamma = (({}^+N, {}^+H, {}^+t, {}^+\Sigma_i, {}^+O, {}^+\succeq_i), Aw_i)$ based on Γ is such that:*

A1 $({}^+N, {}^+H, {}^+t, {}^+\Sigma_i, {}^+O, {}^+\succeq_i)$is a finite extensive game;

A2 $Aw_i : {}^+H_i \to 2^H$ describes i's awareness level at each nonterminal history after which player i moves. For each $h \in {}^+H_i$, $Aw_i(h)$ consists of a set of histories in H and all their prefixes. Intuitively, $Aw_i(h)$ describes the set of histories of Γ that i is aware of at history $h \in {}^+H_i$.

A3 ${}^+N \subseteq N$

A4 if ${}^+t(h) \in {}^+N$, then ${}^+t(h) = t(\bar{h})$, where $\bar{h}$ be the subsequence of h consisting of the actions in h that are also available in Γ. and ${}^+A_h \subseteq A_{\bar{h}}$, intuitively, all the actions available to i at h must also be available to i in the underlying game Γ.

A11 $\{\bar{z}|\ z \in {}^+Z\} \subseteq Z$, i.e. the runs of the game ${}^+\Gamma$ correspond to terminal histories of Γ; moreover for $i \in {}^+N$, $h \in {}^+H_i$, if z is a run in $Aw_i(h)$, then $z \in Z$. i.e. runs of which players are aware are runs of the game Γ upon which ${}^+\Gamma$ is based.

A12 for all $i \in {}^+N$ and runs $z \in {}^+Z$ such that $\bar{z} \in Z$, we have that $z\ {}^+\succeq_i\ \bar{z}$ and $\bar{z}\ {}^+\succeq_i\ z$, i.e. players' preferences are inherited from game Γ upon which ${}^+\Gamma$ is based.

Definition 9. *(Games with awareness). Let* Γ *be a finite extensive game. A game with awareness based on* Γ *is a tuple* $\Gamma^{Aw} = (\mathcal{E}, \Gamma^m, \mathcal{F})$*, where*

- $\mathcal{E}$ *is a countable set of augmented games based on* Γ*, one of the augmented games in* $\mathcal{E}$ *is* Γ^m*;*
- Γ^m *is a distinguished augmented game from the point of view of an omniscient modeler.*
- $\mathcal{F}$ *is a mapping that associates to each augmented game* ${}^+\Gamma \in \mathcal{E}$ *and a history* h *of* ${}^+\Gamma$ *an augmented game* $\Gamma_h \in \mathcal{E}$*.* Γ_h *is the game the player whose turn is to play believes to be the true game when the history is* h*.*

As we can see from the above definition, each player at each history is associated to a game that he believes to be the current game, which can be distinct from the current game being played. ${}^+\Gamma$ represents the point of view of some player at some history. But there is an omniscient modeler who can actually see the game that is being played, whose point of view is Γ^m.

4.2 Representation of games with short sight

The definition below shows that a game with short sight can be represented as a game with awareness where at each choice point players believe to be playing the game induced by their sight.

Definition 10. *(Canonical representation of short sight). Let* $S = (\mathcal{G}^g, s)$ *be a finite prioritized game with short sight with* $\mathcal{G}^g = (N, H, t, \Sigma_i, O, g_i)$*. Let* $\Gamma\lceil_h$ *be the resulting extensive game of* S *for a non-terminal history* h*. The canonical representation of* $(\mathcal{G}^g, s)$ *consists of the tuple*

$$\Gamma^{(\mathcal{G}^g,s)} = (\{\{(\Gamma\lceil_h, Aw_i\lceil_h)|h \in H\}, \Gamma^m\}, \Gamma^m, \mathcal{F})$$

where:

1. Γ^m=$((N, H, t, \Sigma_i, O), \succeq_i), Aw_i)$ *with* $Aw_i(h) = H\lceil_h = s(h)$*, i.e. the modeler knows the structure of the game and the awareness function returns the sight of the players at each point;*

2. For each augmented game ${}^+\Gamma = (\Gamma\lceil_h, Aw_i\lceil_h)$*,* $\Gamma\lceil_h = (N\lceil_h, H\lceil_h, t\lceil_h, \Sigma_i\lceil_h, O\lceil_h, \succeq_i\lceil_h)$ *where* $\succeq_i\lceil_h = \succsim^{g_i} \cap (Z\lceil_h \times Z\lceil_h)$*, and* $Aw_i\lceil_h(h') = Aw_i(h, h') = s(h, h')$*, i.e. preference relation in every augmented game is consistent with the P-graph in its terminal nodes and players' awareness in each augmented game agrees with their sight in the original game;*

3. $\mathcal{F}(\Gamma^m, h) = (\Gamma\lceil_h, Aw_i\lceil_h)$*;*

4. $\mathcal{F}((\Gamma\lceil_h, Aw_i\lceil_h), h') = (\Gamma\lceil_{(h,h')}, Aw_i\lceil_{(h,h')})$*.*

The fourth and fifth item above say that the awareness function coincides with the sight of the players at each decision point.

4.3 Results on equilibrium

In this section, we present the generalized results on equilibrium in games. First, we need to introduce the definition of subgame of prioritized games.

Definition 11. *(Subgames of prioritized games) The subgame of the prioritized game* $\mathcal{G}^g = (N, H, t, \Sigma_i, O, g_i)$ *from history* h *is* $\mathcal{G}^g_h = (N, H|_h, t|_h, \Sigma_i|_h, O|_h, g_i|_h)$*, where* $H|_h$ *is the set of histories* h' *for which* $(h, h') \in H$*;* $t|_h$ *is defined by* $t|_h(h') = t(h, h')$ *for each* $h' \in H|_h$*;* $\Sigma_i|_h$ *consists of elements* $\sigma_i|_h(h') = \sigma_i(h, h')$ *for each* $h' \in H$ *with* $t(h, h') = i$*;* $O|_h : \Pi_{i\in N}\Sigma_i|_h \rightarrow Z|_h$*, where* $Z|_h$ *is the set of* h' *such that* $(h, h') \in Z$*;* $g_i|_h = g_i$*.*

Definition 12. *(Subgame perfect equilibrium) Take a finite prioritized game* $\mathcal{G}^g$*. A strategy profile* σ^* *is a subgame perfect equilibrium if for every player* i *and every nonterminal history* h *for which* $t(h) = i$ *we have that:* $O|_h(\sigma^*_i|_h, \sigma^*_{-i}|_h) \succsim^{g_i} O|_h(\sigma_i, \sigma^*_{-i}|_h)$*, for every strategy* σ_i *available to player* i *in the subgame* $\mathcal{G}^g_h$ *that differs from* $\sigma^*_i|_h$ *only in the action it prescribes after the initial history of* $\mathcal{G}^g_h$*.*

Remark 4. The definition of subgame perfect equilibrium generalizes the normal definition in games. Ours seems more reasonable, since there are occasions when the outcome histories of two strategy profiles are incomparable, resulting in the possibility of both the two strategy profiles are perfect equilibrium. Following this point, we could obtain the generalized definition of sight-compatible subgame perfection.

Definition 13. *(Sight-compatible subgame perfection). Let $S = (\mathcal{G}^g, s)$ be a game with short sight and let $\Gamma\lceil_h$ be the extensive game yielded by s at h. A profile of strategies σ^* is a sight-compatible subgame perfect equilibrium of S if for every player i and every nonterminal history h for which $t(h) = i$ we have: $O\lceil_h(\sigma^*_{-i}\lceil_h, \sigma^*_i\lceil_h) \succeq_i\lceil_h O\lceil_h(\sigma_{-i}\lceil_h, \sigma^*_i\lceil_h)$, for every $\sigma_i \in \Sigma_i\lceil_h$ that differs from σ^*_i only in the action that prescribes at the initial history of $s(h)$.*

Theorem 2. *Every finite horizon game with short sight has a sight-compatible subgame perfect equilibrium.*

4.4 Preference dynamics in representation

We now discuss the preference dynamics in the representation theorem for games with short sight, which is natural and important issues but have rarely been studied yet. We will present some key results through the two theorems below:

Theorem 3. *Priority graph changes in games with short sight result in the changes of preference relation in its canonical representation.*

Theorem 4. *Sight changes in games with short sight give rise to the changes of preference relation in its canonical representation.*

5 Conclusion and Future work

Motivated by real game scenarios, we have proposed a prioritized game model in which players' preference and reasons can be studied altogether. We showed that the incomparable relation between two histories in game is natural and calls for a new modeling. We have generalized the main research results of [2] which are based on linearly ordered priority sequence to the situations which allow for incomparable priorities. We looked at two kinds of dynamics changes in games with short sight, changes in priority graph and changes in sight. We showed that changes of priority in games with short sight result in changes of preference in its canonical representation, and changes of short sight may also lead to preference dynamics in the corresponding games with awareness.

For future directions, we would like to work with prioritized game model and explore how to obtain the existing theories in game theory in our framework. We want to explicitly introduce knowledge or beliefs, and study its relation with player's preference against the traditional quantitative model of games, and investigate how they play a role in real games. Finally, dynamical changes in priorities may behave differently in other scenarios, for instance, repeated games, we are interested in studying them.

References

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Appendix

Theorem 1. For two strategy profiles σ and σ' with $O(\sigma) = h$ and $O(\sigma') = h'$, if h and h' are at the same level of the preference graph, then σ and σ' are indistinguishable.

Proof. If two nodes in the preference graph are at the same level, it must be the case: first, they satisfy or dissatisfy some properties at the same time; second, as to other properties, one of the two nodes satisfies one property but the other node satisfies one property which is incomparable with the former one in the priority graph. In the preference graph of Example 3. node $sp \cap cm \cap ma \cap -ta$ (we represent it by n) and node $sp \cap cm \cap ma \cap -ta$ (we represent it by n') satisfy the same properties sp and cm, but n satisfies ma while n' satisfies ta which is incomparable with ma. So players do not have a clear preference between n and n'. If h is a history in the position n, and h' in n', then h and h' are indistinguishable considering player's preference. Since $O(\sigma) = h$ and $O(\sigma') = h'$, we have that σ is as good as σ', i.e., σ and σ' are indistinguishable. □

Fact 1. From the perspective of equilibrium in games, $h \bowtie^g h'$ can be seen as the same as $h \backsim^g h'$.

This can be obtained directly from Theorem 1.

Fact 2. Let $\mathcal{G}$ be an extensive game form and g $=(\mathcal{H}, \succ)$ a P-graph for $\mathcal{G}$. The relation $\succsim^g$ has the following properties:

1. It is a total pre-order;
2. $\succsim^g$ contains at most $2^{|\mathcal{H}|}$ equivalence classes.

Proof. 1. we have to prove $\succsim^g$ is 1)reflexive, 2)transitive,and 3)total.

1) since $h \sim^g h$ and $h \sim^g h$, by the definition of $\succsim^g$, we know $h \succsim^g h$.

2) for histories h_1,h_2,h_3, assume $h_1 \succsim^g h_2$, $h_2 \succsim^g h_3$, there are four possible cases of the relation between h_1 and h_2 and h_3: i)$h_1 >^g h_2$ and $h_2 >^g h_3$. ii) $h_1 \backsim^g h_2$ and $h_2 >^g h_3$. iii) $h_1 >^g h_2$ and $h_2 \backsim^g h_3$. iv) $h_1 \backsim^g h_2$ and $h_2 \backsim^g h_3$, and either of the four cases will result in $h_1 \succsim^g h_3$

3) for any history h_1 and h_2, there are two cases: i. if they are comparable, then $h1 \gtrsim^g h2$ or $h2 \gtrsim^g h1$, thus $h_1 \succsim^g h_2$; ii. If they are incomparable, then $h_1 \bowtie^g h_2$, which can be seen as $h_1 \backsim^g h_2$ according to Fact 1, thus $h_1 \succsim^g h_2$. So $\succsim^g$ is total.

2. Equivalence classes in $\succsim^g$ are determined by the set of properties in $\mathcal{H}$ that they satisfy, hence by elements of $\wp(\mathcal{H})$. As some of these sets might be empty, $2^{|\mathcal{H}|}$ is an upper bound. □

Theorem 2. Every finite horizon game with short sight has a sight-compatible subgame perfect equilibrium.

Proof. As proved in [2], this theorem could be obtained naturally. □

Theorem 3. Priority graph changes in games with short sight result in the changes of preference relation in its canonical representation.

Proof. According to representation theorem, take a game with short sight $S = (\mathcal{G}^g, s)$ with $\mathcal{G}^g = (N, H, t, \Sigma_i, O, g_i)$, let $\Gamma^{(\mathcal{G}^g,s)} = (\{\{(\Gamma\lceil_h, Aw_i\lceil_h)|\ h \in H\}, \Gamma^m\}, \Gamma^m, \mathcal{F})$ be the canonical representation of S. Then changes of g_i would cause the corresponding preference changes in $\Gamma^{(\mathcal{G}^g,s)}$. Take the operators on two P-graphs for example, we have that:

1. When the parallel composition happens in priority graph, i.e. changes from g_i to $g_{i_1} \parallel g_{i_2}$, the preference relation in $\Gamma^{(\mathcal{G}^g,s)}$ would change from $^+\succeq_i = (^+Z \times {}^+Z) \cap \succsim^{g_i}$ to $^+\succeq_i' = (^+Z \times {}^+Z) \cap \succsim^{g_{i_1} \parallel g_{i_2}}$, where $\succsim^{g_{i_1} \parallel g_{i_2}} = \succsim^{g_{i_1}} \cap \succsim^{g_{i_2}}$.

2. When sequential composition happens in priority graph, i.e. changes from g_i to g_{i_1}; g_{i_2}, the preference relation in $\Gamma^{(\mathcal{G}^g,s)}$ would change from $^+\succeq_i = (^+Z \times {}^+Z) \cap \succsim^{g_i}$ to $^+\succeq_i' = (^+Z \times {}^+Z) \cap \succsim^{g_{i_1};g_{i_2}}$, where $\succsim^{g_{i_1};g_{i_2}} = (\succsim^{g_{i_1}} \cap \succsim^{g_{i_2}}) \cup \succsim^{g^{>}_{i_1}}$. □

Theorem 4. Sight changes in games with short sight give rise to the changes of preference relation in its canonical representation.

Proof. Generally, sight changes could be sorted into two types: one is increase, we define the operator $^{\uparrow}$ to denote it; the other is decrease, the corresponding operator is $^{\downarrow}$.

Take a game with short sight $S = (\mathcal{G}^g, s)$ with $\mathcal{G}^g = (N, H, t, \Sigma_i, O, g_i)$, let $\Gamma^{(\mathcal{G}^g,s)} = (\{\{(\Gamma\lceil_h, Aw_i\lceil_h)| h \in H\}, \Gamma^m\}, \Gamma^m, \mathcal{F})$ be the canonical representation of S. Then changes of s and the corresponding preference changes in $\Gamma^{(\mathcal{G}^g,s)}$ are as follows:

1. When the sight at h increases, i.e. $s(h)^{\uparrow}$, S would change from $(\mathcal{G}^g, s)$ to $(\mathcal{G}^g, s^{\uparrow})$, and the preference relation in $\Gamma^{(\mathcal{G}^g,s)}$ would change from $^+\succeq_i = (^+Z \times {}^+Z) \cap \succsim^{g_i}$ to $^+\succeq_i' = (^+Z^{\uparrow} \times {}^+Z^{\uparrow}) \cap \succsim^{g_i}$, where $^+Z^{\uparrow}$ is the corresponding change of ^+Z caused by the increase of $s(h)$ (according to Fact 3).

2. When the sight at h decreases, i.e. $s(h)^{\downarrow}$, S would change from $(\mathcal{G}^g, s)$ to $(\mathcal{G}^g, s^{\downarrow})$, the preference relation in $\Gamma^{(\mathcal{G}^g,s)}$ would change from $^+\succeq_i = (^+Z \times {}^+Z) \cap \succsim^{g_i}$ to $^+\succeq_i' = (^+Z^{\downarrow} \times {}^+Z^{\downarrow}) \cap \succsim^{g_i}$, where $^+Z^{\downarrow}$ is the corresponding change of ^+Z caused by the decrease of $s(h)$ (according to Fact 3). $\square$

For the limitation of the length, we do not discuss the detail of definition of games with awareness in the main text. Now we make a supplementary illustration through the following Fact.

Fact 4. The augmented game Γ^m and the mapping $\mathcal{F}$ must satisfy a number of consistency conditions. In the following constraints, $M1, M2$ and $M3$ apply to Γ^m. Since the modeler is presumed to be omniscient, these conditions say that modeler is aware of all the players and moves of the underlying game. $C1$ is the constraint on $\mathcal{F}$:

$M1$ $N^m = N$

$M2$ $A \subseteq A^m$ and $\{\bar{z} | z \in Z^m\} = Z$.

$M3$ If $t^m(h) \in N$ then $A^m_h = A_{\bar{h}}$, i.e. the modeler is aware of the possible courses of the events;

$C1$ Suppose that for $^+\Gamma \in \mathcal{E}$, $h \in {}^+H$, $^+t(h) = i$, and $\mathcal{F}(^+\Gamma, h) = \Gamma_h$, we have that $\{\bar{h'} \mid h' \in H_h\} = Aw_i(h)$, where H_h is the set of histories in Γ_h.

$C1$ guarantees that the set of histories of the underlying game player i is aware of is exactly the set of histories of the underlying game that appear in Γ_h.