Computational Epistemology for Quantifiers

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VIIth Tbilisi Symposium
October 5, 2007

Outline

1. Problems

2. Quantifiers
   - Quantifiers of type $\langle 1 \rangle$
   - Quantifiers of type $\langle 1, 1 \rangle$

3. Computational Epistemology

4. Identifiability

5. General question

PLAN

1. PROBLEMS
2. QUANTIFIERS
   - Quantifiers of type $\langle 1 \rangle$
   - Quantifiers of type $\langle 1, 1 \rangle$
3. COMPUTATIONAL EPISTEMOLOGY
4. IDENTIFIABILITY
5. GENERAL QUESTION

PROBLEMS

- Epistemological properties of quantifiers.
- Their influence on NL comprehension.
- Linking them to learnability features.
- Compare notions of decidability and identifiability.

PLAN

QUANTIFIERS
LINDSTRÖM DEFINITION

DEFINITION

A monadic generalized quantifier of type $\underbrace{(1, \ldots, 1)}_{n}$ is a class Q of structures of the form $\mathbf{M} = (M, A_1, \ldots, A_n)$, where A_i is a subset of M. Additionally, Q is closed under isomorphism.

Q OF TYPE $\langle 1 \rangle$

MONOTONICITY

DEFINITION

$\mathsf{Q}_{\mathbf{M}}$ is MON$\uparrow$ iff: if $A \subseteq A' \subseteq M$, then $\mathsf{Q}_{\mathbf{M}}(A)$ implies $\mathsf{Q}_{\mathbf{M}}(A')$.

DEFINITION

$\mathsf{Q}_{\mathbf{M}}$ is MON$\downarrow$ iff: if $A' \subseteq A \subseteq M$, then $Q_{\mathbf{M}}(A)$ implies $Q_{\mathbf{M}}(A')$.

Q of type $\langle 1 \rangle$

Extendability

Definition

Q of type $\langle 1 \rangle$ satisfies *EXT* iff for all models **M** and $\mathbf{M}'$:
$A \subseteq M \subseteq M'$ implies $\mathsf{Q}_{\mathbf{M}}(A) \iff \mathsf{Q}_{\mathbf{M}'}(A)$.

Quantifiers of type $\langle 1, 1 \rangle$

Preconditions

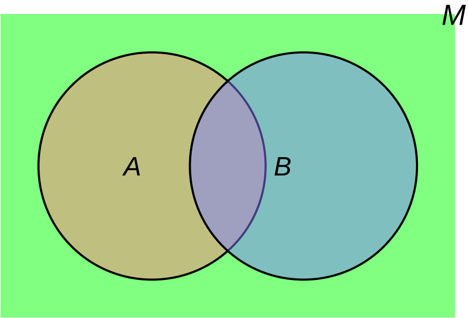
Restriction to CE quantifiers.

Definition

Let Q be of type $\langle 1, 1 \rangle$.
Then for all $\mathbf{M}$, $\mathbf{M}'$, all $A, B \subseteq M$, and $A', B' \subseteq M'$:
(ISOM) If $(M, A, B) \cong (M', A', B')$, then $Q_{\mathbf{M}}(A, B) \Leftrightarrow Q_{\mathbf{M}'}(A', B')$.
(CONS) $Q_{\mathbf{M}}(A, B) \Leftrightarrow Q_{\mathbf{M}}(A, A \cap B)$.
(EXT) If $M \subseteq M'$, then $Q_{\mathbf{M}}(A, B) \Leftrightarrow Q_{\mathbf{M}'}(A, B)$.

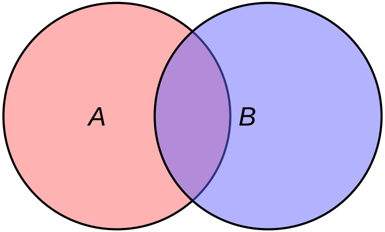
CE Quantifiers

(ISOM) If $(M, A, B) \cong (M', A', B')$, then $Q_M(A, B) \Leftrightarrow Q_{M'}(A', B')$



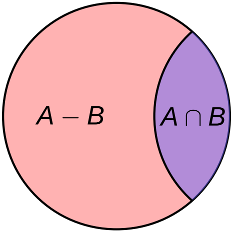
CE Quantifiers - EXT

(EXT) If $M \subseteq M'$, then $\mathsf{Q}_{\mathsf{M}}(A, B) \Leftrightarrow \mathsf{Q}_{\mathsf{M'}}(A, B)$



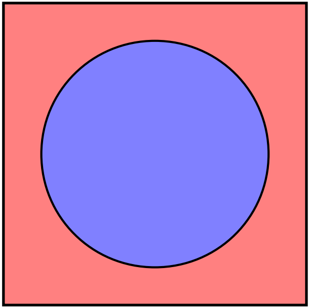
CE Quantifiers - CONS

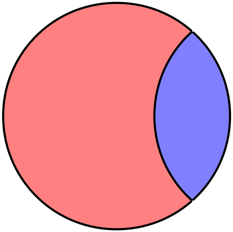
(CONS) $\mathsf{Q}_{\mathbf{M}}(A, B) \Leftrightarrow \mathsf{Q}_{\mathbf{M}}(A, A \cap B)$



CE QUANTIFIERS

The scope we are interested in for both $\langle 1 \rangle$ and $\langle 1, 1 \rangle$ cases.





Q OF TYPE $\langle 1, 1 \rangle$

MONOTONICITY

DEFINITION

Q of type $\langle 1, 1 \rangle$ is MON$\uparrow$ iff:
If $A \subseteq M$ and $B \subseteq B' \subseteq M$, then $\mathsf{Q}_{\mathbf{M}}(A, B) \Rightarrow \mathsf{Q}_{\mathbf{M}}(A, B')$.

Q of type $\langle 1, 1 \rangle$

Persistence

Definition

Q of type $\langle 1, 1 \rangle$ is PER iff:
If $A \subseteq A' \subseteq M$ and $B \subseteq M$, then $\mathrm{Q}_{\mathbf{M}}(A, B) \Rightarrow \mathrm{Q}_{\mathbf{M}}(A', B)$.

Examples

Determiner	MON $\uparrow$	EXT (for $\langle 1 \rangle$)	PER (for $\langle 1, 1 \rangle$)
All	+	-	-
No	-	-	-
Some	+	+	+
At least n	+	+	+
At most n	-	+	-
Exactly n	-	+	-

TABLE: Quantifiers and their properties

MONOTONICITY & LINGUISTICS

- Does monotonicity influence NL comprehension?
- Does monotonicity influence NL learning?
- Monotonicity and inference patterns (B. Geurts).
- Proposal: focus on persistence.

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Logic of reliable inquiry - Kevin Kelly

- Inspired by learning theory.
- Similar framework.
- Verification/falsification in computational setting.

Logic of reliable inquiry - Kevin Kelly

ε — infinite string of data;

$\varepsilon|n$ — finite initial segment of ε through the position $n-1$;

h — hypothesis;

C — correctness relation;

$C(\varepsilon, h)$ iff h is correct w.r.t. ε;

α — an assessment method;

OUT conjectures 1, 0, !.

Certainty in reliable inquiry

Definition

α produces b with certainty on (h, ε) iff there is an n s.t.:

1. $\alpha(h, \varepsilon|n) = !$, and
2. $\alpha(h, \varepsilon|n + 1) = b$, and
3. for each $m < n$, $\alpha(h, \varepsilon|m) \neq !$.

Certainty in reliable inquiry

Definition

α verifies h with certainty on ε (with respect to C) iff α produces 1 with certainty on $(h, \varepsilon) \Leftrightarrow C(\varepsilon, h)$.

Definition

α refutes h with certainty on ε (with respect to C) iff α produces 0 with certainty on $(h, \varepsilon) \Leftrightarrow \neg C(\varepsilon, h)$.

Definition

Decidability with certainty is simply verifiability and refutability with certainty at the same time.

Examples

- At least six bikes are broken. - Verifiable with certainty
- An even number of bikes is broken. - Verifiable in the limit

Epistemological properties of Q o.t. $\langle 1 \rangle$

- $1 - 1$ enumeration of elements of the universe.
- Assignment of χ_A to each of them.
- Infinite sequence of 0s and 1s.
- In each step checking if finite initial segment satisfies a hypothesis (quantifier sentence).

Epistemological properties of Q o.t. $\langle 1 \rangle$

Proposition

Let Q be FO quantifier of type $\langle 1 \rangle$.
Q is MON $\uparrow$ and EXT iff it is verifiable with certainty.

Proposition

Let Q be FO quantifier of type $\langle 1 \rangle$.
$\neg$Q is verifiable with certainty iff Q is falsifiable with certainty.

Epistemological properties of Q o.t. $\langle 1, 1\rangle$

Proposition

Let Q be FO CE quantifier of type $\langle 1, 1\rangle$.
Q is persistent iff it is verifiable with certainty.

Proposition

Let Q be FO CE quantifier of type $\langle 1, 1\rangle$.
$\neg$Q is falsifiable with certainty iff Q is verifiable with certainty.

Examples

Determiner	verifiable	falsifiable	MON $\uparrow$	EXT (for $\langle 1 \rangle$)	PER (for $\langle 1, 1 \rangle$)
All	-	+	+	-	-
No	-	+	-	-	-
Some	+	-	+	+	+
At least n	+	-	+	+	+
At most n	-	+	-	+	-
Exactly n	-	-	-	+	-

Table: Quantifiers and their properties

PLAN

1 PROBLEMS

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3 COMPUTATIONAL EPISTEMOLOGY

4 IDENTIFIABILITY

5 GENERAL QUESTION

IDENTIFIABILITY GAME

- Class of objects is chosen (e.g. class of grammars).
- Player 1 picks out one object from the class (e.g. G).
- Player 1 generates positive instances of this object, repetitions allowed (e.g. words from a language of G).
- Player 2 knows about the class, but he does not know which object is chosen.
- Player 2 has to guess which object Player 1 has in mind.

LEARNING THE SEMANTICS OF NATURAL LANGUAGE

IDENTIFIABILITY FROM TEXT IN USE

- Class of quantifiers is chosen.
- Player 1 picks one of them (Q)
- Player 2 is presented finite worlds in which Q is true.
- Player 2 has to identify Q.

Number triangle representation

Assuming CE, we can represent all relevant models in the form of number triangle.

$$
\begin{array}{ccccccccc}
 & & & & (0,0) & & & & \\
 & & & (1,0) & & (0,1) & & & \\
 & & (2,0) & & (1,1) & & (0,2) & & \\
 & (3,0) & & (2,1) & & (1,2) & & (0,3) & \\
(4,0) & & (3,1) & & (2,2) & & (1,3) & & (0,4) \\
\ldots & & & & & & & & \ldots
\end{array}
$$

Number triangle representation

- Graphic representation of a class of CE quantifiers.
- In particular: PER.

$$\begin{array}{ccccccccccc}
 & & & & & - & & & & & \\
 & & & & - & & + & & & & \\
 & & & - & & + & & + & & & \\
 & & - & & + & & + & & + & & \\
 & - & & + & & + & & + & & + & \\
\ldots & & & & & & & & & & \ldots
\end{array}$$

Number triangle representation

- Graphic representation of a class of CE quantifiers.
- In particular: PER.

```
        -
      -   +
    -   +   +
  -   +   +   +
-   +   +   +   +
...             ...
```

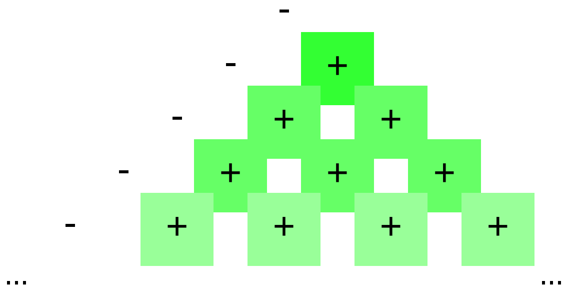
Number triangle representation

- Graphic representation of a class of CE quantifiers.
- In particular: PER.

```
            -
          -   +
        -   +   +
      -   +   +   +
    -   +   +   +   +
...                   ...
```

Number triangle representation

- Graphic representation of a class of CE quantifiers.
- In particular: PER.



TIEDE'S RESULT

THEOREM

The class of FO PER Q is identifiable from text.

Plan

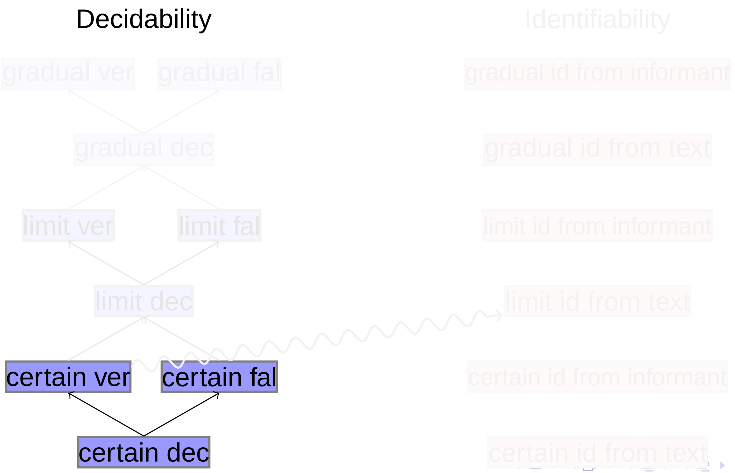
General question

Relation between Ver/Fal hierarchy and identifiability

Decidability

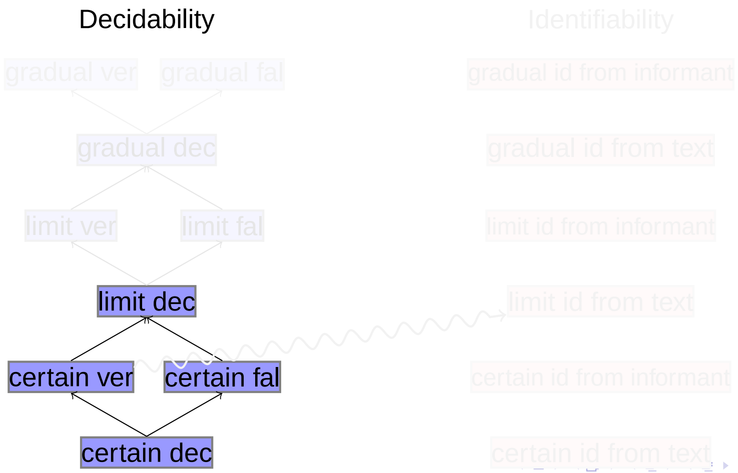
Identifiability

General question
Relation between Ver/Fal hierarchy and identifiability



General question

Relation between Ver/Fal hierarchy and identifiability



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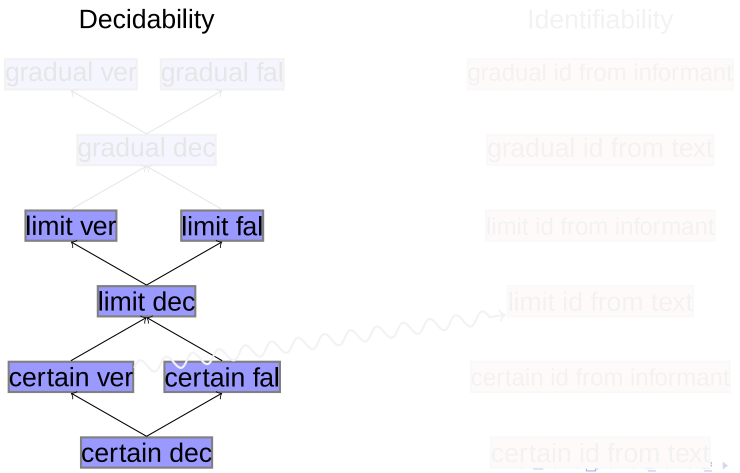

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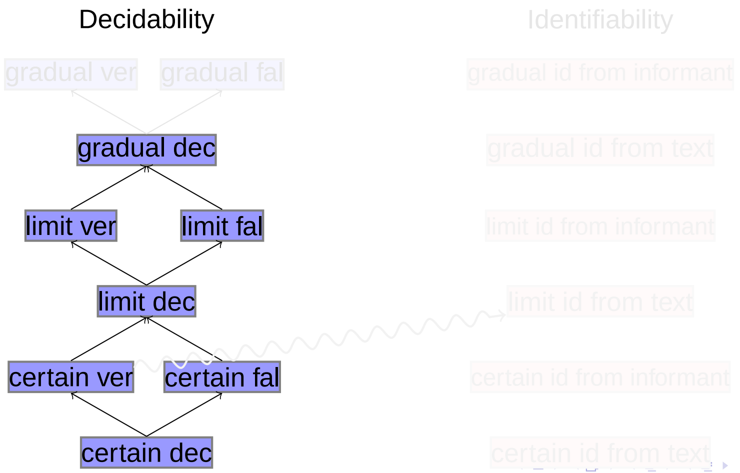

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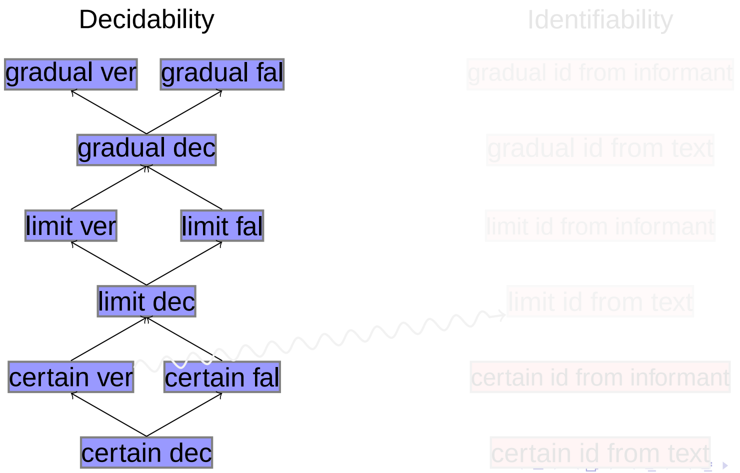
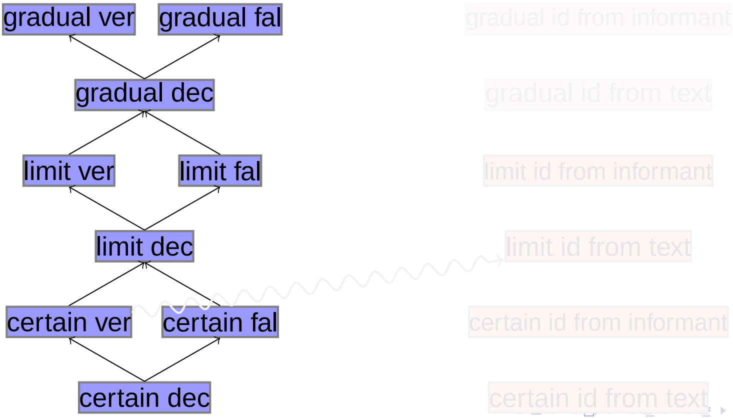


General question

Relation between Ver/Fal hierarchy and identifiability

Decidability | Identifiability



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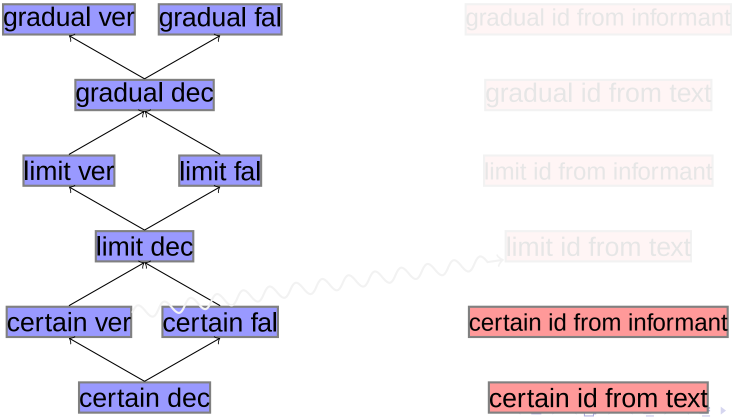
Relation between Ver/Fal hierarchy and identifiability

Decidability

Identifiability



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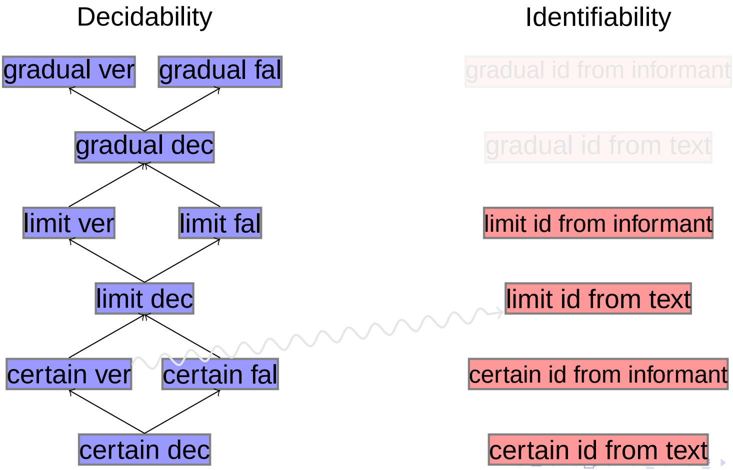

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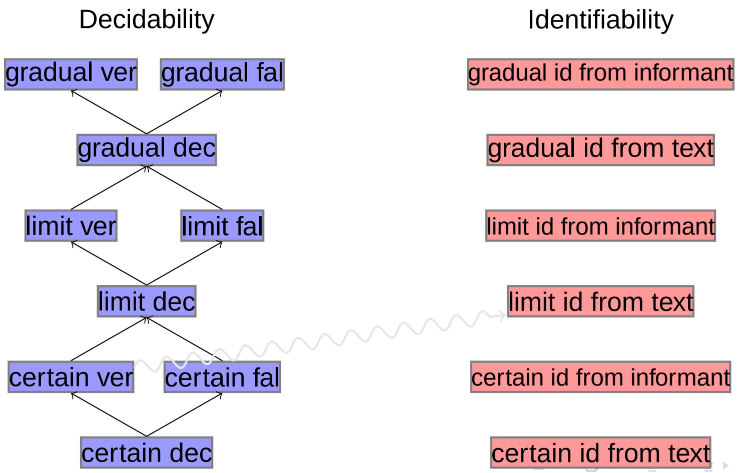

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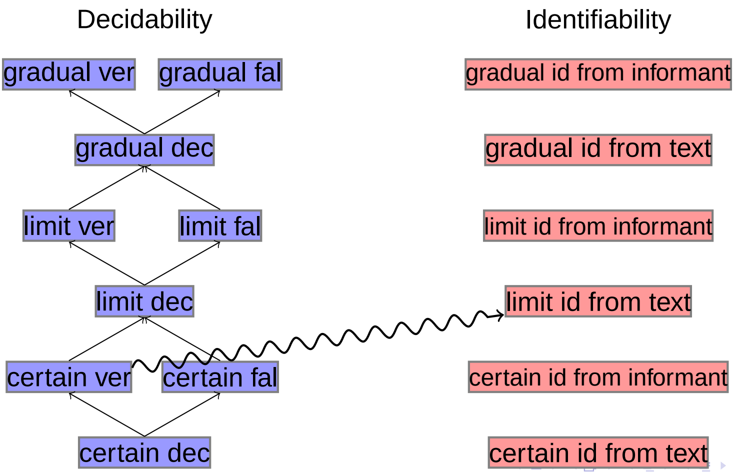

Conclusions and future work

- Epistemological role of monotonicity - additional explanation.
- Verification less difficult than falsification?
- Check connections between persistence and comprehension.
- Investigate relationship between identifiability and decidability: learning of NL semantics; new conditions of identifiability.

TH∀NK YOU!