# The Characterization of Finite Identifiability

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**Theorem 1** *A class $\mathcal{L}$ is finitely identifiable from positive data if and only if a definite finite tell-tale of Li is uniformly computable for any index $i$, that is, there exists an effective procedure that on input $i$ produces all elements of a definite finite tell-tale of $L_i$ and then halts.*

**Proof**

[⇒]

Suppose the class $\mathcal{L}$ is finitely identifiable from positive data. Then there exists a learner $M$ that finitely identifies $\mathcal{L}$. Take $L_i$ and a text $t$ for $L_i$. A definite finite tell-tale of $L_i$ is uniformly computable in the following way.

$$D(i) = \mathit{content}(t[n]) \text{ s.t. } M(t[n]) = j \text{ and } L_j = L_i.$$

Since $M$ finitely identifies the class $\mathcal{L}$, $D(i)$ always exists and is computable. Now, we show by contradiction that the output of this procedure, say $D$, is a definite finite tell-tale of $L_i$. Suppose $D$ is not a definite finite tell-tale of $L_i$. Clearly, $D$ is a finite subset of $L_i$. Therefore, there exists an index $j$ such that $L_i \neq L_j$ and $D \subseteq L_j$. Since $M$ infers $L_i$ from $t$ such that there is an $n$ with $\mathit{content}(t[n]) = D$, it follows that $M$ can not infer $L_j$ from a positive presentation $t^j$ of $L_j$ such that $t^j[n] = t[n]$. This contradicts the assumption.

[⇐]

Suppose a definite finite tell-tale of $L_i$ is uniformly computable for any index $i$, and we denote by $D(i)$ the result of computation. The class $\mathcal{L}$ is finitely identifiable by the following learner $M$. $M$ constructs set $T$, that is initially empty. At each step $n$ $M$ adds the newly observed data into $T$ and checks if $D(j) \subseteq T$ for $0 \leq j \leq n$. Once such is found $M$ outputs $j$ and stops.

The problem: $D(j) \subseteq T$? is decidable since $D(j)$ and $T$ are explicitly given finite sets.

1. The output is a correct guess by the definition of definite tell-tale set.

2. The learner gives an answer after a finite time.

□