Formal Learning Theory
Homework 1

(due on Friday, February 15th at 1pm)

February 8th, 2013

Ex. 1 A text t is called *ascending* if $t_n \leq t_{n+1}$ for all $n \in \mathbb{N}$; t is called *strictly ascending* if $t_n < t_{n+1}$ for all $n \in \mathbb{N}$.

(a) Let L be a finite language such that $card(L) \geq 2$. How many ascending texts are there for L?

(b) Let L be an infinite language. How many strictly ascending texts are there for L?

Ex. 2 Let $\mathcal{L}$ be a class containing all co-doubletons, i.e., $\mathcal{L} = \{\mathbb{N} - \{n, m\} \mid n, m \in \mathbb{N} \text{ and } n \neq m\}$. Let $\mathcal{L}'$ be a class containing all co-dubletons and all co-singletons, i.e., $\mathcal{L}' = \{\mathbb{N} - \{k\} \mid k \in \mathbb{N}\} \cup \{\mathbb{N} - \{n, m\} \mid n, m \in \mathbb{N} \text{ and } n \neq m\}$.

(a) Show that $\mathcal{L}$ is identifiable.

(b) Show that $\mathcal{L}'$ is not identifiable.

Ex. 3 Let M identify $\mathcal{L}$. Show that for each $\tau \in \mathbb{N}^*$ with $content(\tau) \subseteq L$ for some $L \in \mathcal{L}$ there is a $\sigma \in \mathbb{N}^*$ such that $\tau^\wedge\sigma$ is a locking sequence for M and L.

Ex. 4 [*] Let $\mathcal{L} = \{\{1, 2\}, \mathbb{N}\}$. Define learner M in the following way:

$$M(t[n+1]) = \begin{cases} \{1,2\} & \text{if } content(t[n+1]) \subseteq \{1,2\}; \\ \mathbb{N} & \text{otherwise, except when } t_n = 4 \text{ and } t_{n+1} = 3 \\ & \text{and this has never happened before (i.e., there is} \\ & \text{no } m < n \text{ such that } t_m = 4 \text{ and } t_m + 1 = 3), \\ & \text{then also } M(t[n+1]) = \{1,2\}. \end{cases}$$

(a) Show M identifies $\mathcal{L}$.

(b) Take $t = \langle 1, 2, 3, 4, \ldots \rangle$, a text for $\mathbb{N}$, show that no initial segment of t is a locking sequence for M and $\mathbb{N}$.

(c) Explain why the above does not contradict the Locking Sequence Theorem and Ex. 3.