# Formal Learning Theory
# Homework 4

(due on Friday, March 8th at 1pm)

March 1st, 2013

**Ex. 1** For any $k \in \mathbb{N}^+$, we use $\mathcal{L}^k$ to denote the class of all unions of up to $k$ languages from $\mathcal{L}$, i.e., $\mathcal{L}^k = \{L_{j_1} \cup \ldots \cup L_{j_k} \mid L_{j_1}, \ldots, L_{j_k} \in \mathcal{L}\}$. Let $\mathcal{L}$ be a uniformly recursive family and $k \in \mathbb{N}^+$. Show that if $\mathcal{L}$ is of finite elasticity, then $\mathcal{L}^k$ is of finite elasticity.

**Ex. 2** Give an example of an effectively identifiable family $\mathcal{L}$ such that $\mathcal{L}$ does not have characteristic sets.

**Ex. 3** Give an example of a uniformly recursively enumerable family that has characteristic sets but is not effectively identifiable.

**Ex. 4** Let $\mathcal{L} = \{L_i \mid i \in \mathbb{N}\}$ be uniformly recursive family that has a family of characteristic sets $(S_i)_{i\in\mathbb{N}}$. Show that there exists a family of characteristic sets $(T_i)_{i\in\mathbb{N}}$ for $\mathcal{L}$ and a recursive function $F(n,i)$ such that $T_i = \bigcup_{i\in\mathbb{N}} T_i^n$, where $T_i^n = F(n,i)$.

**Ex. 5** Let $\mathcal{L}$ contain the following languages:

1. $L_i = \{i, i^2, i^3, \ldots, i^i\}$, for $i \in \mathbb{N}^+$;
2. $L_0 = \mathbb{N}$.

Show that $\mathcal{L}$ is effectively identifiable (use a proof strategy **different than showing a witness learner**).