Formal Learning Theory
Homework 5

(due on Friday, March 15th at 1pm)

March 8th, 2013

Ex. 1 Recall the following theorem (proved in Lecture 9).

Theorem 1 *$\mathcal{L} = \{K \cup \{x\} \mid x \notin K\}$ can be given as a uniformly r.e. class.*

Proof Let $k_0, k_1, k_2, \ldots$ be en enumeration of K. Let n be a specific number not in K. There are lots of those, e.g. any index of the everywhere undefined function will do. Then below there is a uniform enumeration of $\mathcal{L}$, given by uniform enumerations of the L_i.

Start each L_i by the number i and continue with $k_0, k_1, k_2, \ldots$. You do this until i appears as k_m. If it does you put n in instead of k_m and continue after with $k_{m+1}, k_{m+2}, \ldots$. there are two possibilities:

1. i is not in K. Then i will never occur as k_m and L_i will be an enumeration of $K \cup \{i\}$.
2. i is in K. Then it will occur as k_m. Then n will be substituted, and L_i will be an enumeration of $K \cup \{n\}$.

It is clear that each $K \cup \{x\}$ for $x \notin K$ is represented in the enumeration, and that each member of the enumeration is some $K \cup \{x\}$ for $x \notin K$. □

In the enumeration, $K \cup \{n\}$ will occur many times. The question of the exercise is, give a 1-1 enumeration of $\mathcal{L}$. *Hint*: You may assume as true that $\overline{K}$ contains an infinite r.e. subset. A question we do not know the answer to: what happens if you replace K by a simple set S (an r.e. set such that $\overline{S}$ does not contain an infinite r.e. subset).

Ex. 2

(a) Give two effectively identifiable uniformly recursive families of languages $\mathcal{L}_1$ and $\mathcal{L}_2$ such that: $\mathcal{L}_1 \uplus \mathcal{L}_2$ is not identifiable, where $\mathcal{L}_1 \uplus \mathcal{L}_2 = \{L_1 \cup L_2 \mid L_1 \in \mathcal{L}_1, L_2 \in \mathcal{L}_2\}$.

(b) Give two effectively identifiable uniformly recursive families of languages $\mathcal{L}_1$ and $\mathcal{L}_2$ such that: $\mathcal{L}_1 \Cap \mathcal{L}_2 = \{L_1 \cap L_2 \mid L_1 \in \mathcal{L}_1, L_2 \in \mathcal{L}_2\}$ is not identifiable.

Ex. 3 Finish the proof of Theorem 3 in the handout of Lecture 9, i.e., show that $\mathcal{L}$ defined in the proof cannot be identified by a recursive function conservative on $\mathcal{L}$ (with respect to any index).

Ex. 4 **Definition 1** *Take $k \in \mathbb{N}$. A learner M identifies a class $\mathcal{L}$ with k-mind changes just in case M identifies $\mathcal{L}$, and for each $L \in \mathcal{L}$ and each text t for L there exists no more than k steps $n \in \mathbb{N}$ such that $M(t[n]) \neq M(t[n+1])$.*

(a) Show that the class $\mathcal{L} = \{\{n, k\} \mid n, k \in \mathbb{N},\ n \neq k\}$ is identifiable with 1-mind change.

(b) Show that the class of all finite sets is not identifiable with n-mind changes, for any $n \in \mathbb{N}$.