Formal Learning Theory
MoL Course, Spring 2013

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Lecture 1
February 5$^{\text{th}}$ 2013

A Motivating Example

An inductive inference game of sets and numbers.

A Motivating Example

An inductive inference game of sets and numbers.

$\{\quad 2, 3, 4, 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{1, 2, 3, \quad 5, \ldots\}$
$\ldots$

1

An inductive inference game of sets and numbers.

$\{\quad 2, 3, 4, 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{1, 2, 3, \quad 5, \ldots\}$
$\ldots$

1, 3

An inductive inference game of sets and numbers.

$\{\quad 2, 3, 4, 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{1, 2, 3, \quad 5, \ldots\}$
$\ldots$

1, 3, 4

A Motivating Example

An inductive inference game of sets and numbers.

$\{\quad 2, 3, 4, 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{1, 2, 3, \quad 5, \ldots\}$
$\ldots$

1, 3, 4, 2

A Motivating Example

An inductive inference game of sets and numbers.

$\{\quad 2, 3, 4, 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{1, 2, 3, \quad 5, \ldots\}$
$\ldots$

1, 3, 4, 2, 6

A Motivating Example

An inductive inference game of sets and numbers.

$\{\quad 2, 3, 4, 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{1, 2, 3, \quad 5, \ldots\}$
$\ldots$

1, 3, 4, 2, 6, 7

A Motivating Example

An inductive inference game of sets and numbers.

$\{\quad 2, 3, 4, 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{1, 2, 3, \quad 5, \ldots\}$
$\ldots$

1, 3, 4, 2, 6, 7, 8

A Motivating Example

An inductive inference game of sets and numbers.

$\{\ 2, 3, 4, 5, \ldots\}$
$\{1, \ 3, 4, 5, \ldots\}$
$\{1, 2, \ 4, 5, \ldots\}$
$\{1, 2, 3, \ 5, \ldots\}$
$\ldots$

$$1,\ 3,\ 4,\ 2,\ 6,\ 7,\ 8,\ \ldots$$

A Motivating Example: Questions

1. Are you confident? What would make you change your guess?
2. What was your "guessing rule"?
3. How do you like winning if at least one of your guess is correct?
4. And if you succeed to make a right guess and never change your mind after that? How many wrong guesses could you make under this condition?

A Motivating Example: Questions

1. Assume that I'll give you all and only truthful clues. What would be the guessing rule to win according to the last rule?
2. Add $\{1, 2, 3, 4, 5, \ldots\}$. Is your guessing rule still good?
3. While keeping $\{1, 2, 3, 4, 5, \ldots\}$ in, assume that I'll give you all and only truthful clues, and I'll guarantee they are ordered increasingly. Can you win the game?
4. Now, remove $\{1, 2, 3, 4, 5, \ldots\}$. You get only one guess—would you object to this winning condition?

1960s: The Beginning of Formal Learning Theory

- Hillary Putnam (1965). Trial and error predicates and the solution to a problem of Mostowski.
- E. Mark Gold (1967). Language identification in the limit.
- Ray Solomonoff (1964). A formal theory of inductive inference.

What is the Course About

The problem of induction
and related issues in epistemology and philosophy of science.

Problems Addressed

- Language Learning/Grammar Inference
- Scientific Inquiry
- Fallible Knowledge
- Reliable Learning
- Computable Learning and AI

Outline

Practical Information about the Course

Formal Learning Theory: Frameworks Overview
- Language Learning
- Function Learning
- Model-theoretic Learning

Language Learning Paradigm

General Information

- Credits: 6 ects
- Grading: 60% weekly homework, 40% final exam
- Timetable: Tuesdays 15-17 (room G2.13) and Fridays 13-15 (room A1.14)
- Website: http://www.ninagierasimczuk.com/flt2013
- Contact: nina.gierasimczuk@gmail.com, d.h.j.dejongh@uva.nl

Homework

- Assignment published on Friday
- Due on following Friday before the class starts (a week after)
- Preferred format: LaTeX $\rightarrow$ PDF
- Teaching assistant: Zoé Christoff (http://zoechristoff.com/)

Exam

- Time: Wednesday, March 27th 2013, 13.00-15.45
- Place: SP 904, room A1.04

Outline

Learning Paradigm

1. Possible realities.
2. Hypotheses.
3. Information accessible to the learner.
4. Learner.
5. Success criterion.

Language Learning Paradigm

Also known as Set Learning and Numerical Paradigm

1. Possible realities:

   Sets of numbers
2. Hypotheses:

   Some names of sets
3. Information accessible to the learner:

   Sequences of numbers
4. Learner:

   Function that takes a sequence and outputs a hypothesis
5. Success criterion:

   After finite number of outputs the answers stabilize on a correct answer

- Daniel Osherson, Michael Stob, and Scott Weinstein (1986 and 1999). Systems that Learn.
- Eric Martin and Daniel Osherson (1998). Elements of Scientific Inquiry, Chapter 2.
- Steffen Lange and Thomas Zeugmann (1995). A Guided Tour Across the Boundaries of Learning Recursive Languages.
- Steffen Lange, Thomas Zeugmann, and Sandra Zilles (2008). Learning indexed families of recursive languages from positive data: A survey.

Function Learning Paradigm

Also known as Learning of Functional Languages

1. Possible realities:

   Functions

2. Hypotheses:

   Names of functions

3. Information accessible to the learner:

   Sequences of pairs (argument, value)

4. Learner:

   Function that takes a sequence and outputs a hypothesis

5. Success criterion:

   After finite number of outputs the answers stabilize on a correct answer

Thomas Zeugmann and Sandra Zilles (2008). Learning recursive functions: A survey.

Kevin Kelly (1996). The Logic of Reliable Inquiry.

1. Possible realities:

   Models of a given signature
2. Hypotheses:

   First order sentences
3. Information accessible to the learner:

   Sequences of atomic formulas and negations thereof
4. Learner:

   Function that takes a sequence and outputs a hypothesis
5. Success criterion:

   After finite number of outputs the answers stabilize on a correct answer

Eric Martin and Daniel Osherson (1998). Elements of Scientific Inquiry, Chapter 3.

Eric Martin and Daniel Osherson (1998). Belief revision in the service of scientific discovery.

1. Possible realities:

   Possible worlds

2. Hypotheses:

   Sets of possible worlds

3. Information accessible to the learner:

   Sequences of propositions

4. Learner:

   Function that takes a sequence and outputs a proposition

5. Success criterion:

   After finite number of outputs the answers stabilize on a proposition that is a singleton of the actual world

Nina Gierasimczuk (2010). Knowing One's Limits. Logical Analysis of Inductive Inference.

Alexandru Baltag, Nina Gierasimczuk, and Alexandru Baltag (2011). Belief Revision as a Truth-Tracking Process.

- Hypotheses are systematic descriptions of possible realities.
- They are sometimes captured as “naming system”.
- The hypotheses are finite descriptions of sets.
- E.g., Turing machines, grammars, natural numbers, logical formulas.

- In interesting cases the data available at a given step presents only partial information about a possible reality.
- The character of data is determined by the setting, e.g. in language learning one might consider only positive or positive and negative information about a possible reality.
- In the basic setting data presented to the learner is arbitrary, in some paradigms the learner can request particular information.

- Finite identifiability.
- Identifiability in the limit.
- Gradual identifiability.

We will fix the success criterion to be:

after a finite time the answers of the learner stabilize on correct answer.

Outline

Definition

Let $\mathbb{N}$ stand for natural numbers. We call any $L \subseteq \mathbb{N}$ a language. Then $\mathcal{L} = (L_i)_{i \in \mathbb{N}}$ is a class of languages.

Definition

By a *text* t of L we mean an infinite sequence of elements from L enumerating all and only the elements from L (allowing repetitions).

Notation

Definition

We will use the following notation:

- t_n is the n-th element of t;
- $t[n]$ is the sequence $(t_0, t_2, \ldots, t_{n-1})$;
- $content(t)$ is the set of elements that occur in t;
- Let $\mathbb{N}^*$ be the set of all finite sequences over $\mathbb{N}$. If $\alpha, \beta \in \mathbb{N}^*$, and $\alpha = \langle x_0, \ldots, x_n \rangle$ and $\beta = \langle y_0, \ldots, y_m \rangle$ then by $\alpha^\wedge\beta$ we mean the concatenation of α and β, i.e., $\langle x_0, \ldots, x_n, y_0, \ldots, y_m \rangle$;
- $M : \mathbb{N}^* \to \mathbb{N}$ is a learning function, a map from finite data sequences to hypotheses.

Identifiability in the Limit

Definition

Learning function M:

1. identifies $L_i \in \mathcal{L}$ in the limit on t iff for co-finitely many m, $M(t[m]) = i$;
2. identifies $L_i \in \mathcal{L}$ in the limit iff it identifies L_i in the limit on every t for L_i;
3. identifies $\mathcal{L}$ in the limit iff it identifies in the limit every $L_i \in \mathcal{L}$.

A class $\mathcal{L}$ is identifiable in the limit iff there is a learning function that identifies $\mathcal{L}$ in the limit.

Example

Let $\mathcal{L}_1 = \{L_i \mid i \in \mathbb{N} - \{0\}\}$, where $L_n = \{1, \ldots, n\}$.

EXAMPLE

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$\mathcal{L}_1$ is identifiable in the limit by the following function $M : \mathbb{N}^* \rightarrow \mathbb{N}$:

$$M(t[n]) = \max(content(t[n])).$$

Some Examples

Example

Let $\mathcal{L}_2 = \{L_i \mid i \in \mathbb{N}\}$, where $L_0 = \mathbb{N}$ and for $n \geq 1$, $L_n = \{1, \ldots, n\}$.

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$\mathcal{L}_2$ is not identifiable in the limit.

Example

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$\mathcal{L}_2$ is not identifiable in the limit.

Argument

To show that this is the case, let us assume that there is a function M that identifies $\mathcal{L}_2$. We will construct a text, t on which M fails:
t starts by enumerating $\mathbb{N}$ in order: $0, 1, 2, \ldots$
if at a number k learner M decides it is L_0, t starts repeating k indefinitely.
This means t is a text for L_k.
As soon as M decides it is L_k we continue with $k+1, k+2, \ldots$, so t will become a text for L_0, etc.
This shows that there is a text for a set from $\mathcal{L}_2$ on which M fails.

Example

Let $\mathcal{L}_4 = \{L_n \mid L_n = \mathbb{N} - \{n\}, n \in \mathbb{N}\}$.

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Let $\mathcal{L}_4 = \{L_n \mid L_n = \mathbb{N} - \{n\}, n \in \mathbb{N}\}$.

$\mathcal{L}_4$ is identifiable in the limit by the learning function $M : \mathbb{N}^* \to \mathbb{N}$:

$$M(t[n]) = \min(\mathbb{N} - content(t[n])).$$

The End
of Lecture 1