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Bargaining Under Strategic Uncertainty

Amanda Friedenberg

Extremely Extremely Preliminary

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Bargaining				

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Bargaining				

Key Feature of Many Applications:

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Key Feature of Many Applications:

- Employment Contracts
- Trials and Arbitration
- Sovereign Debt
- War
- Legislative Bargaining
- etc.



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Key Feature of Many Applications:

- Employment Contracts
- Trials and Arbitration
- Sovereign Debt
- War
- Legislative Bargaining
- etc.

Important Behavioral Feature:

• Failure to Reach Immediate Agreement

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A Source of Bargaining Impasse

Strategic Uncertainty



Set-Up

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Set-Up

Necessity

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The Nature of Strategic Uncertainty 000

A Source of Bargaining Impasse

Strategic Uncertainty

Two Concerns:

Too Many Predictions?



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A Source of Bargaining Impasse

Strategic Uncertainty

- Too Many Predictions?
 - Sophisticated Reasoning about Strategic Uncertainty?

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A Source of Bargaining Impasse

Strategic Uncertainty

- Too Many Predictions?
 - Sophisticated Reasoning about Strategic Uncertainty?
- 2 Too Few Predictions

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A Source of Bargaining Impasse

Strategic Uncertainty

- Too Many Predictions?
 - Sophisticated Reasoning about Strategic Uncertainty?
- 2 Too Few Predictions
 - Sophisticated Reasoning about Strategic Uncertainty

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A Source of Bargaining Impasse

Strategic Uncertainty

- Too Many Predictions?
 - Sophisticated Reasoning about Strategic Uncertainty?
- 2 Too Few Predictions
 - Sophisticated Reasoning about Strategic Uncertainty
 - Limit predictions to rule out impasse?

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A Source of Bargaining Impasse

Strategic Uncertainty

- Too Many Predictions?
 - Sophisticated Reasoning about Strategic Uncertainty?
- 2 Too Few Predictions
 - Sophisticated Reasoning about Strategic Uncertainty
 - Limit predictions to rule out impasse?

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Forward Induction Reasoning: Kohlberg, 1981

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Forward Induction Reasoning: Kohlberg, 1981

• Rationalize Past Behavior when Possible



Forward Induction Reasoning: Kohlberg, 1981

- Rationalize Past Behavior when Possible
- Formalization: Battigalli and Siniscalchi (2002)
 - Rationality and Common Strong Belief of Rationality

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• Belief about How Others Play the Game

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• Belief about How Others Play the Game: b_i^1

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Forward Induction Reasoning: Kohlberg, 1981

- Rationalize Past Behavior when Possible
- Formalization: Battigalli and Siniscalchi (2002)
 - Rationality and Common Strong Belief of Rationality
- Belief about How Others Play the Game: b_i^1
 - Rationality: Maximize (Conditional) SEU
 - Property of (s_i, b_i^1)

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Forward Induction Reasoning: Kohlberg, 1981

- Rationalize Past Behavior when Possible
- Formalization: Battigalli and Siniscalchi (2002)

Necessity

• Rationality and Common Strong Belief of Rationality

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- Belief about How Others Play the Game: b_i^1
 - Rationality: Maximize (Conditional) SEU
 - Property of (s_i, b_i^1)
- Belief about Play and b_{-i}^1

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Forward Induction Reasoning: Kohlberg, 1981

- Rationalize Past Behavior when Possible
- Formalization: Battigalli and Siniscalchi (2002)

Necessity

• Rationality and Common Strong Belief of Rationality

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- Belief about How Others Play the Game: b_i^1
 - Rationality: Maximize (Conditional) SEU
 - Property of (s_i, b_i^1)
- Belief about Play and b_{-i}^1 : b_i^2

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- Rationality and Common Strong Belief of Rationality
- Belief about How Others Play the Game: b_i^1
 - Rationality: Maximize (Conditional) SEU
 - Property of (s_i, b_i^1)
- Belief about Play and b_{-i}^1 : b_i^2
 - Assign Probability 1 to event "Rational" when Possible

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- Rationalize Past Behavior when Possible
- Formalization: Battigalli and Siniscalchi (2002)

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- Rationality and Common Strong Belief of Rationality
- Belief about How Others Play the Game: b_i^1
 - Rationality: Maximize (Conditional) SEU
 - Property of (s_i, b_i^1)
- Belief about Play and b_{-i}^1 : b_i^2
 - Assign Probability 1 to event "Rational" when Possible

• And so on.

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Lessons from Finite Games



Lessons from Finite Games

Belief Dependent Concept



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Belief Dependent Concept

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Belief Dependent Concept

Formally:

• Type Structure: Hierarchies of Beliefs about the Play of the Game

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Belief Dependent Concept

Formally:

• Type Structure: Hierarchies of Beliefs about the Play of the Game

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• Behavioral Predictions can change with Type Structure

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Belief Dependent Concept

Formally:

- Type Structure: Hierarchies of Beliefs about the Play of the Game
- Behavioral Predictions can change with Type Structure
- In Particular:
 - "Rich" Type Structure: Extensive-Form Rationalizability

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- Battigalli and Siniscalchi, 2002
- "Small" Type Structure: Disjoint Prediction



Lessons from Finite Games

Belief Dependent Concept



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Belief Dependent Concept

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Belief Dependent Concept

Conceptually:

• How Does a Player Update His Hypothesis when Surprised?

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Belief Dependent Concept

- How Does a Player Update His Hypothesis when Surprised?
- Tension between Giving up on:
 - (a) Hypothesis that other player is rational vs.
 - (b) Hypothesis about the other player's beliefs

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Belief Dependent Concept

- How Does a Player Update His Hypothesis when Surprised?
- Tension between Giving up on:
 - (a) Hypothesis that other player is rational vs.
 - (b) Hypothesis about the other player's beliefs
- Forward Induction Reasoning: Give up on (b)

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Belief Dependent Concept

- How Does a Player Update His Hypothesis when Surprised?
- Tension between Giving up on:
 - (a) Hypothesis that other player is rational vs.
 - (b) Hypothesis about the other player's beliefs
- Forward Induction Reasoning: Give up on (b)
- Small Type Structures:
 - Limit the ability to Give up on (b)
Belief Dependent Concept

Conceptually:

- How Does a Player Update His Hypothesis when Surprised?
- Tension between Giving up on:
 - (a) Hypothesis that other player is rational vs.
 - (b) Hypothesis about the other player's beliefs
- Forward Induction Reasoning: Give up on (b)
- Small Type Structures:
 - Limit the ability to Give up on (b)
- What Small Type Structures are Meant to Capture
 - Restrictions on Players' Beliefs
 - Game Described as Part of a Bigger Context

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Sophisticated Reasoning about Strategic Uncertainty



Sophisticated Reasoning about Strategic Uncertainty

A. Restriction on Players' Beliefs





- A. Restriction on Players' Beliefs
 - No On Path Strategic Uncertainty



- A. Restriction on Players' Beliefs
 - No On Path Strategic Uncertainty
 - Along the Path of Play: Correct Beliefs
 - About Terminal Node



- A. Restriction on Players' Beliefs
 - No On Path Strategic Uncertainty
 - Along the Path of Play: Correct Beliefs
 - About Terminal Node
 - If surprised:
 - Must have incorrect beliefs



- A. Restriction on Players' Beliefs
 - No On Path Strategic Uncertainty
 - Along the Path of Play: Correct Beliefs
 - About Terminal Node
 - If surprised:
 - Must have incorrect beliefs
 - Connections:
 - Self-Confirming Equilibrium
 - Applications



Sophisticated Reasoning about Strategic Uncertainty

- A. Restriction on Players' Beliefs
 - No On Path Strategic Uncertainty
 - Along the Path of Play: Correct Beliefs
 - About Terminal Node
 - If surprised:
 - Must have incorrect beliefs
 - Connections:
 - Self-Confirming Equilibrium
 - Applications

B. Forward Induction Reasoning



Sophisticated Reasoning about Strategic Uncertainty

- A. Restriction on Players' Beliefs
 - No On Path Strategic Uncertainty
 - Along the Path of Play: Correct Beliefs
 - About Terminal Node
 - If surprised:
 - Must have incorrect beliefs
 - Connections:
 - Self-Confirming Equilibrium
 - Applications

B. Forward Induction Reasoning

• Co-player satisfies No On Path Strategic Uncertainty

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Sophisticated Reasoning about Strategic Uncertainty

- A. Restriction on Players' Beliefs
 - No On Path Strategic Uncertainty
 - Along the Path of Play: Correct Beliefs
 - About Terminal Node
 - If surprised:
 - Must have incorrect beliefs
 - Connections:
 - Self-Confirming Equilibrium
 - Applications

B. Forward Induction Reasoning

- Co-player satisfies No On Path Strategic Uncertainty
- Limits ability to rationalize co-player's past behavior

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Behavioral	Implicat	ions		

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Behavioral	Implicat	cions		

Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

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Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

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Necessity

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Behavioral	Implicat	cions		

Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

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Necessity



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Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

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- Necessity
 - Not an "Anything Goes" Result



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Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

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- Necessity
 - Not an "Anything Goes" Result
 - With Deadline: Never delay until the last period



Introduction	Set-Up	Necessity	Sufficiency	The Nature of Strategic Uncertainty
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Behavioral	Implicat	ions		

Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

- Necessity
 - Not an "Anything Goes" Result
 - With Deadline: Never delay until the last period
 - With Deadline: Limits on delay to penultimate period

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Sufficiency

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Rehavioral Implications						

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Main Theorem

Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

- Necessity
 - Not an "Anything Goes" Result
 - With Deadline: Never delay until the last period
 - With Deadline: Limits on delay to penultimate period

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- Limits on the negotiated outcomes
- Sufficiency

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Behavioral	Implicat	ions		

Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

- Necessity
 - Not an "Anything Goes" Result
 - With Deadline: Never delay until the last period
 - With Deadline: Limits on delay to penultimate period

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- Limits on the negotiated outcomes
- Ø Sufficiency
 - Can have impasse

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Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

- Necessity
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- Limits on the negotiated outcomes
- Ø Sufficiency
 - Can have impasse
 - Depends on the deadline (if there is any)

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Behavioral	Implicat	ions		

Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

- Necessity
 - Not an "Anything Goes" Result
 - With Deadline: Never delay until the last period
 - With Deadline: Limits on delay to penultimate period

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- Limits on the negotiated outcomes
- ② Sufficiency
 - Can have impasse
 - Depends on the deadline (if there is any)
 - Depends on Bargainers' patience

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Bargaining	Game:	B		

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Bargaining	Game:	B		
Timeline				

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	0P Barg	ainer 1 Offer	rs: $x \in [0, 1]$		

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	0P Barga	ainer 1 Offe	rs: $x \in [0, 1]$		
	0R Barga	ainer 2 Choo	oses: A or R		

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Ba	rgaining	Game:	B			
	Timeline					
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	0R Barga	iner 2 Cho	oses: A or R			
	•	lf A				

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Ba	rgaining Game:	B				
	Timeline					
	0P Bargainer 1 Offers: $x \in [0, 1]$					
	0R Bargainer 2 Chooses: A or R					
	• If A: (x, 1	-x, 0)				

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Ba	rgaining Game:	B				
	Timeline					
	0P Bargainer 1 Offers: $x \in [0, 1]$					
	0R Bargainer 2 Chooses: A or R					
	 If A: (x,1 - If R 	- x, 0)				

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Ba	rgaining Gam	e: <i>B</i>				
	Timeline					
	0P Bargainer 1	Offers: $x \in [0, 1]$				
	0R Bargainer 2 Chooses: A or R					
	If A: (xIf R	, 1 – <i>x</i> , 0)				
	1P Bargainer 2	Offers: $y \in [0, 1]$				

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Ba	rgaining	Game:	B			
	Timeline					
	0P Bargainer 1 Offers: $x \in [0, 1]$					
	0R Bargainer 2 Chooses: A or R					
	• •	If A: (<i>x</i> ,1 – If R	- x, 0)			
	1P Barga	ainer 2 Offe	rs: $y \in [0,1]$			
	1R Barga	ainer 1 Choo	oses: A or R			

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	0P Bargainer 1 Offe	rs: $x \in [0, 1]$		
	0R Bargainer 2 Cho	oses: A or R		
	If A: (x,1 -If R	- x, 0)		
	1P Bargainer 2 Offe	rs: $y \in [0, 1]$		
	1R Bargainer 1 Cho	oses: A or R		
	● If A: (1 – y	', y, 1)		

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	Timeline					
	0P Bargainer 1 Offers: $x \in [0, 1]$					
	0R Bargainer 2 Chooses: A or R					
	 If A: (x, 1 − x, 0) If R 					
	1P Bargain	er 2 Offers:	$y \in [0,1]$			
	1R Bargain	er 1 Choos	es: A or R			
	● If ● If	A: (1 – y, y R:	/,1)			

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Set-Up Necessity The Nature of Strategic Uncertainty 000000 Bargaining Game: \mathcal{B} Timeline 0P Bargainer 1 Offers: $x \in [0, 1]$ 0R Bargainer 2 Chooses: A or R • If A: (x, 1 - x, 0)• If R 1P Bargainer 2 Offers: $y \in [0, 1]$ 1R Bargainer 1 Chooses: A or R • If A: (1 - y, y, 1)• If R: (0,0,1), if N=1 is the quitting period

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Set-Up Necessity The Nature of Strategic Uncertainty 000000 Bargaining Game: \mathcal{B} Timeline 0P Bargainer 1 Offers: $x \in [0, 1]$ 0R Bargainer 2 Chooses: A or R • If A: (x, 1 - x, 0)• If R 1P Bargainer 2 Offers: $y \in [0, 1]$ 1R Bargainer 1 Chooses: A or R • If A: (1 - y, y, 1)• If R: (0,0,1), if N=1 is the quitting period Quitting Period: N

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- N Finite: Deadline
- N Infinite: No Deadline

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	Timeline				
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	0R Barga	ainer 2 Cho	oses: A or R		
	9 9	If A: (<i>x</i> ,1 – If R	- x,0)		
	1P Barga	ainer 2 Offe	rs: $y \in [0, 1]$		
	1R Bargainer 1 Chooses: A or R				
	9	If A: (1 – y If R:	r, y, 1)		
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- N Finite: Deadline
- N Infinite: No Deadline

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	Timeline				
0P Bargainer 1 Offers: $x \in [0, 1]$					
	0R Barga	ainer 2 Choo	oses: A or R		
	•	If A: (x ,1 – If R	- x,0)		
	1P Barga	ainer 2 Offe	rs: $y \in [0, 1]$		
	1R Bargainer 1 Chooses: A or R				
	•	If A: (<mark>1 – y</mark> If R:	, y, 1)		
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- N Finite: Deadline
- N Infinite: No Deadline
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If R | - x ,0) | | |
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	 Quitt 	ing Period:	N		

- N Finite: Deadline
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	Timeline					
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	0R Barga	iner 2 Choc	oses: A or R			
	•	lf A: (<i>x</i> ,1 – lf R	x,0)			
	1P Barga	iner 2 Offer	s: $y \in [0, 1]$			
	1R Bargainer 1 Chooses: A or R					
	•	If A: (1 – y If R:	(y, 1)			
	2P					
	• Quitti	ing Period:	N			

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- N Finite: Deadline
- N Infinite: No Deadline
- Payoffs: Share of z in period n gives $\delta^n z$

Modeling Strategic Uncertainty



What we will Need:





What we will Need:

 b_i^1 Bargainer *i*'s beliefs about how other plays





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What we will Need:

- b_i^1 Bargainer *i*'s beliefs about how other plays
- b_i^2 Bargainer *i*'s beliefs about b_{-i}^1 ,



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Modeling Strategic Uncertainty

What we will Need:

- b_i^1 Bargainer *i*'s beliefs about how other plays
- b_i^2 Bargainer *i*'s beliefs about b_{-i}^1 ,
 - etc.



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Modeling Strategic Uncertainty

What we will Need:

- b_i^1 Bargainer *i*'s beliefs about how other plays
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What we will Really Need:



Modeling Strategic Uncertainty

What we will Need:

- b_i^1 Bargainer *i*'s beliefs about how other plays
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 - etc.

What we will Really Need:

• Bargainer *i* may begin the game with one hypothesis

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Modeling Strategic Uncertainty

What we will Need:

- b_i^1 Bargainer *i*'s beliefs about how other plays
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- May be forced to revise beliefs
 - If other Bargainer plays differently



What we will Need:

- b_i^1 Bargainer *i*'s beliefs about how other plays
- b_i^2 Bargainer *i*'s beliefs about b_{-i}^1 ,
 - etc.

What we will Really Need:

• Bargainer *i* may begin the game with one hypothesis

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- May be forced to revise beliefs
 - If other Bargainer plays differently
- Hierarchies of Conditional Beliefs about the Play

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Type Str	uctures			

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For each Player *i*:

1 Type Set: T_i

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Ingredients of a Type Structure

- **1** Type Set: T_i
- **2** Belief Map: β_i

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Ingredients of a Type Structure

- **1** Type Set: T_i
- **2** Belief Map: β_i
 - Map type t_i to belief on $S_{-i} \times T_{-i}$

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Ingredients of a Type Structure

- Type Set: T_i
- **2** Belief Map: β_i
 - Map type t_i to belief on $S_{-i} \times T_{-i}$
 - To system of beliefs on $S_{-i} imes T_{-i}$

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For each Player *i*:

- Type Set: T_i
- **2** Belief Map: β_i
 - Map type t_i to belief on $S_{-i} \times T_{-i}$
 - To system of beliefs on $S_{-i} imes T_{-i}$
 - One belief for each information set

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Type Str	uctures			

For each Player *i*:

- **1** Type Set: T_i
- **2** Belief Map: β_i
 - Map type t_i to belief on $S_{-i} \times T_{-i}$
 - To system of beliefs on $S_{-i} imes T_{-i}$
 - One belief for each information set
 - Satisfy rules of conditional probability if possible

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 How to Think of the Objects on the Table



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How to Think of the Objects on the Table

Description

Epistemic Game: $(\mathcal{B}, \mathcal{T})$

- Bargaining Game
- O Type Structure



How to Think of the Objects on the Table

Description

Epistemic Game: $(\mathcal{B}, \mathcal{T})$

- Bargaining Game
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Induces Set of States: (s_1, t_1, s_2, t_2)



How to Think of the Objects on the Table

Description Epistemic Game: (B, T) a Bargaining Game a Type Structure Induces Set of States: (s₁, t₁, s₂, t₂)

Epistemic Conditions: Restriction on the Set of States

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How to Think of the Objects on the Table

Description

- Epistemic Game: $(\mathcal{B}, \mathcal{T})$
 - Bargaining Game
 - O Type Structure
- Induces Set of States: (s_1, t_1, s_2, t_2)

Epistemic Conditions: Restriction on the Set of States

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No On Path Strategic Uncertainty

How to Think of the Objects on the Table

Description

Epistemic Game: $(\mathcal{B}, \mathcal{T})$

- Bargaining Game
- O Type Structure

Induces Set of States: (s_1, t_1, s_2, t_2)

Epistemic Conditions: Restriction on the Set of States

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- No On Path Strategic Uncertainty
- Porward Induction Reasoning

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Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible



Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

Steps to Formalization:



Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

Steps to Formalization:

- Actionality:
 - Property of (*s_i*, *t_i*)



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Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

Steps to Formalization:

- Actionality:
 - Property of (*s_i*, *t_i*)
- Strong Belief: "Thinking"

Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

Steps to Formalization:

- Actionality:
 - Property of (*s_i*, *t_i*)
- Strong Belief: "Thinking"
 - Strong Belief is a Property of a Type t_i

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Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

Steps to Formalization:

- Rationality:
 - Property of (*s_i*, *t_i*)
- Strong Belief: "Thinking"
 - Strong Belief is a Property of a Type t_i
 - Assign probability 1 to E_{-i} , if $E_{-i} \cap [S_{-i}(h) \times T_{-i}] \neq \emptyset$

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Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

Steps to Formalization:

- Actionality:
 - Property of (s_i, t_i)

Rationality and Strong Belief of Rationality

- Strong Belief is a Property of a Type t_i
- Assign probability 1 to E_{-i} , if $E_{-i} \cap [S_{-i}(h) \times T_{-i}] \neq \emptyset$

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Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

Steps to Formalization:

- Rationality:
 - Property of (s_i, t_i)
 - Set of Rational Strategy-Type Pairs of $i: R_i^1$
- Rationality and Strong Belief of Rationality
 - Strong Belief is a Property of a Type t_i
 - Assign probability 1 to E_{-i} , if $E_{-i} \cap [S_{-i}(h) \times T_{-i}] \neq \emptyset$

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Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

Steps to Formalization:

- Rationality:
 - Property of (s_i, t_i)
 - Set of Rational Strategy-Type Pairs of $i: R_i^1$
- Rationality and Strong Belief of Rationality
 - Strong Belief is a Property of a Type t_i
 - Assign probability 1 to R^1_{-i} , if $R^1_{-i} \cap [S_{-i}(h) \times T_{-i}] \neq \emptyset$

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Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

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Forward Induction Reasoning

Forward Induction

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No On Path Strategic Uncertainty

Say there is **no on path strategic uncertainty at a state** (s_1, t_1, s_2, t_2) if, for each information set along the path of play induced by (s_1, s_2) , t_1 (resp. t_2) assigns probability 1 to reaching the terminal node, viz. z^* , induced by (s_1, s_2) .

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No On Path Strategic Uncertainty

Say there is **no on path strategic uncertainty at a state** (s_1, t_1, s_2, t_2) if, for each information set along the path of play induced by (s_1, s_2) , t_1 (resp. t_2) assigns probability 1 to reaching the terminal node, viz. z^* , induced by (s_1, s_2) .

Rephrase:

- Event $\mathbb{Z}_{-i}[s_1, s_2]$:
 - Event the terminal node associated with (s_1, s_2) , viz. z^* , is reached, when s_i is played

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• Each t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$

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Level 1

• Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 imes R_2^1$

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Level 1

• Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$
- t_i strongly believes R_{-i}^1

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Level 1

• Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$
- t_i strongly believes R_{-i}^1
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$

Introduction Set-Up Necessity Sufficiency The Nature of Strategic Uncertainty Constrained Conditions Conditions Conditions Conditions

Level 1

• Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

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Level 1

• Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
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Level 3

Introduction Set-Up Necessity Sufficiency The Nature of Strategic Uncertainty Conditions Epistemic Conditions Conditions Conditions Conditions

Level 1

• Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

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Level 3

• Survives Level 2: $(s_1, t_1, s_2, t_2) \in R_1^2 \times R_2^2$

Introduction Set-Up Necessity Sufficiency The Nature of Strategic Uncertainty Constrained Conditions Conditions Conditions Conditions

Level 1

• Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2: $R_1^2 \times R_2^2$

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

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Level 3

- Survives Level 2: $(s_1, t_1, s_2, t_2) \in R_1^2 \times R_2^2$
- t_i strongly believes R_{-i}^2

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Level 1

• Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2: $R_1^2 \times R_2^2$

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

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Level 3

- Survives Level 2: $(s_1, t_1, s_2, t_2) \in R_1^2 \times R_2^2$
- t_i strongly believes R_{-i}^2

Level 4 ...

Level 1

• Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 imes R_2^1$

Level 2: $R_1^2 \times R_2^2$

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

Level 3

- Survives Level 2: $(s_1, t_1, s_2, t_2) \in R_1^2 \times R_2^2$
- t_i strongly believes R_{-i}^2

Level 4 ...

Forward Induction Reasoning Under No On Path Strategic Uncertainty

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Bounds on	Delay			

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Bounds or	n Delay			

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Bounds on outcomes will come from two levels of reasoning:

- Rationality
- Strong Belief of Rationality
- No On Path Strategic Uncertainty

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Bounds on	Delay			

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Bounds on outcomes will come from two levels of reasoning:

- Rationality
- Strong Belief of Rationality
- No On Path Strategic Uncertainty

Examples:

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Bounds on	Delay			

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Bounds on outcomes will come from two levels of reasoning:

- Rationality
- Strong Belief of Rationality
- No On Path Strategic Uncertainty

Examples:

Two Period Example: No Delay

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Bounds on	Delay			

Bounds on outcomes will come from two levels of reasoning:

- Rationality
- Strong Belief of Rationality
- No On Path Strategic Uncertainty

Examples:

- Two Period Example: No Delay
- **2** Three Period Example: If Delay then $(\delta, 1 \delta, 1)$

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Bounds on	Delay			

Bounds on outcomes will come from two levels of reasoning:

- Rationality
- Strong Belief of Rationality
- No On Path Strategic Uncertainty

Examples:

- Two Period Example: No Delay
- **2** Three Period Example: If Delay then $(\delta, 1 \delta, 1)$

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 $\bullet\,$ Only Happen if δ sufficiently large

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The Two Period Deadline

Suppose Delay

• Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$

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The Two Period Deadline

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Along Path: 2 Proposes

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The Two Period Deadline

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Along Path: 2 Proposes
- t_2^* Strongly Believes 1 is Rational

The Two Period Deadline

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Along Path: 2 Proposes
- t_2^* Strongly Believes 1 is Rational
 - When Propose: Continues to believe 1 is Rational

The Two Period Deadline

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Along Path: 2 Proposes
- t_2^* Strongly Believes 1 is Rational
 - When Propose: Continues to believe 1 is Rational
- (s_2^*, t_2^*) Rational and Strongly Believes Rational:
 - 2 offers to take the full pie and expects 1 to Accept

The Two Period Deadline

Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Along Path: 2 Proposes
- t_2^* Strongly Believes 1 is Rational
 - When Propose: Continues to believe 1 is Rational
- (s_2^*, t_2^*) Rational and Strongly Believes Rational:
 - 2 offers to take the full pie and expects 1 to Accept

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- No On Path Strategic Uncertainty:
 - t_1^* begins the game believing: (0, 1, 1)

The Two Period Deadline

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Along Path: 2 Proposes
- t_2^* Strongly Believes 1 is Rational
 - When Propose: Continues to believe 1 is Rational
- (s_2^*, t_2^*) Rational and Strongly Believes Rational:
 - 2 offers to take the full pie and expects 1 to Accept
- No On Path Strategic Uncertainty:
 - t_1^* begins the game believing: (0, 1, 1)
- Strong Belief of Rationality
 - t_1^* begins the game believing: 2 Accepts any $x < 1 \delta$ upfront

The Two Period Deadline

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Along Path: 2 Proposes
- t_2^* Strongly Believes 1 is Rational
 - When Propose: Continues to believe 1 is Rational
- (s_2^*, t_2^*) Rational and Strongly Believes Rational:
 - 2 offers to take the full pie and expects 1 to Accept
- No On Path Strategic Uncertainty:
 - t_1^* begins the game believing: (0, 1, 1)
- Strong Belief of Rationality
 - t_1^* begins the game believing: 2 Accepts any $x < 1 \delta$ upfront
- 1 would strictly prefer to offer some $x < 1 \delta$ split upfront

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Three Period Deadline						

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Introduction	Set-Up	Necessity	Sufficiency	The Nature of Strategic Uncertainty		
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Suppose Delay

• Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$
Introduction	Set-Up	Necessity	Sufficiency	The Nature of Strategic Uncertainty	
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Three Period Deadline					

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$

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Three Period Deadline						

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:

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- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:
 - Continues to believe $(x^*, 1 x^*, 1)$

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- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:
 - Continues to believe $(x^*, 1 x^*, 1)$
 - Continues to Believe 1 is Rational

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Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:
 - Continues to believe $(x^*, 1 x^*, 1)$
 - Continues to Believe 1 is Rational
 - Believes 1 will Accept any Offer $y < 1 \delta$

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Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:
 - Continues to believe $(x^*, 1 x^*, 1)$
 - Continues to Believe 1 is Rational
 - Believes 1 will Accept any Offer $y < 1 \delta$

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• So: $1-x^* \ge 1-\delta$ or $\delta \ge x^*$

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Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:
 - Continues to believe $(x^*, 1 x^*, 1)$
 - Continues to Believe 1 is Rational
 - Believes 1 will Accept any Offer $y < 1 \delta$

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• So:
$$1 - x^* \ge 1 - \delta$$
 or $\delta \ge x^*$

• When 1 Accepts $1-\delta$

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Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:
 - Continues to believe $(x^*, 1 x^*, 1)$
 - Continues to Believe 1 is Rational
 - Believes 1 will Accept any Offer $y < 1 \delta$

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• So:
$$1 - x^* \ge 1 - \delta$$
 or $\delta \ge x^*$

- When 1 Accepts 1δ
 - Continues to Believe 2 is Rational

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Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:
 - Continues to believe $(x^*, 1 x^*, 1)$
 - Continues to Believe 1 is Rational
 - Believes 1 will Accept any Offer $y < 1 \delta$
 - So: $1-x^* \ge 1-\delta$ or $\delta \ge x^*$
- When 1 Accepts $1-\delta$
 - Continues to Believe 2 is Rational
 - Believes 2 will Accept any third-period Offer z < 1

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Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:
 - Continues to believe $(x^*, 1 x^*, 1)$
 - Continues to Believe 1 is Rational
 - Believes 1 will Accept any Offer $y < 1 \delta$
 - So: $1-x^* \ge 1-\delta$ or $\delta \ge x^*$
- When 1 Accepts 1δ
 - Continues to Believe 2 is Rational
 - Believes 2 will Accept any third-period Offer z < 1

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• So: $\delta x^* \ge \delta^2 z$ for all z < 1 or $x^* \ge \delta$

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Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$
- When 2 Proposes $1 x^*$:
 - Continues to believe $(x^*, 1 x^*, 1)$
 - Continues to Believe 1 is Rational
 - Believes 1 will Accept any Offer $y < 1 \delta$
 - So: $1 x^* \ge 1 \delta$ or $\delta \ge x^*$
- When 1 Accepts 1δ
 - Continues to Believe 2 is Rational
 - Believes 2 will Accept any third-period Offer z < 1

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• So: $\delta x^* \ge \delta^2 z$ for all z < 1 or $x^* \ge \delta$

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Three Period Deadline Revisited

Suppose Delay

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Three Period Deadline Revisited

Suppose Delay

• Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$

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Three Period Deadline Revisited

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- At 1's initial node: continues to believe outcome

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Three Period Deadline Revisited

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- At 1's initial node: continues to believe outcome
 - Expected payoffs δx^*

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Three Period Deadline Revisited

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- At 1's initial node: continues to believe outcome
 - Expected payoffs δx^*
- At 1's initial node: Continues to Believe 2 is Rational

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Three Period Deadline Revisited

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 x^*, 1)$
- At 1's initial node: continues to believe outcome
 - Expected payoffs δx^*
- At 1's initial node: Continues to Believe 2 is Rational
 - Believes 2 will Accept any Offer $y < 1 \delta$

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Three Period Deadline Revisited

Suppose Delay

• Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$

• At 1's initial node: continues to believe outcome

- Expected payoffs δx^*
- At 1's initial node: Continues to Believe 2 is Rational
 - Believes 2 will Accept any Offer $y < 1 \delta$

• So:
$$\delta x^* \geq 1 - \delta$$
 or $x^* \geq rac{1-\delta}{\delta}$

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Three Period Deadline Revisited

Suppose Delay

• Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$

• At 1's initial node: continues to believe outcome

• Expected payoffs δx^*

• At 1's initial node: Continues to Believe 2 is Rational

• Believes 2 will Accept any Offer $y < 1 - \delta$

• So:
$$\delta x^* \geq 1 - \delta$$
 or $x^* \geq rac{1-\delta}{\delta}$

• Upper Bound: δ

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Three Period Deadline Revisited

Suppose Delay

• Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$

• At 1's initial node: continues to believe outcome

• Expected payoffs δx^*

• At 1's initial node: Continues to Believe 2 is Rational

• Believes 2 will Accept any Offer $y < 1 - \delta$

• So:
$$\delta x^* \geq 1 - \delta$$
 or $x^* \geq rac{1-\delta}{\delta}$

- Upper Bound: δ
- Lower Bound Bound:

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Three Period Deadline Revisited

Suppose Delay

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• So:
$$\delta x^* \geq 1 - \delta$$
 or $x^* \geq rac{1-\delta}{\delta}$

- Upper Bound: δ
- Lower Bound Bound:
 - No Incentive to Delay till Deadline: δ

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• No Incentive to Settle Upfront: $\frac{1-\delta}{\delta}$

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Three Period Deadline Revisited

Suppose Delay

• Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$

• At 1's initial node: continues to believe outcome

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• At 1's initial node: Continues to Believe 2 is Rational

• Believes 2 will Accept any Offer $y < 1 - \delta$

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- Upper Bound: δ
- Lower Bound Bound:
 - No Incentive to Delay till Deadline: δ

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• No Incentive to Settle Upfront: $\frac{1-\delta}{\delta}$

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Three Period Deadline Revisited

Suppose Delay

• Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$

• At 1's initial node: continues to believe outcome

• Expected payoffs δx^*

• At 1's initial node: Continues to Believe 2 is Rational

• Believes 2 will Accept any Offer $y < 1 - \delta$

• So:
$$\delta x^* \geq 1 - \delta$$
 or $x^* \geq rac{1-\delta}{\delta}$

- Upper Bound: δ
- Lower Bound Bound:
 - No Incentive to Delay till Deadline: δ

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- No Incentive to Settle Upfront: $\frac{1-\delta}{\delta}$
- Delay only if sufficiently patient

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No Deadli	ne: Nece	ssitv		

Fix some epistemic game (B, T) with no deadline. Suppose that, at $(s_1^*,t_1^*,s_2^*,t_2^*)$

- each player is rational
- each player strongly believes the other player is rational, and
- there is no on path strategic uncertainty.

Then, (s_1^*, s_2^*) induces an outcome $(x^*, 1 - x^*, n^*)$ with $x^* \in [\underline{x}_{n^*}, \overline{x}_{n^*}]$.

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- each player is rational
- each player strongly believes the other player is rational, and
- there is no on path strategic uncertainty.

Then, (s_1^*, s_2^*) induces an outcome $(x^*, 1 - x^*, n^*)$ with $x^* \in [\underline{x}_{n^*}, \overline{x}_{n^*}]$.

$$\underline{x}_{n^*} = \frac{1-\delta}{\delta^{n^*}} \quad \text{and} \quad \overline{x}_{n^*} = \begin{cases} 1 - \frac{\delta(1-\delta)}{\delta^{n^*}} & \text{if } n^* \ge 1\\ 1 & \text{if } n^* = 0. \end{cases}$$

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Illustration of Possible Outcomes: No Deadline





One Period of Delay: Lower Bound

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One Period of Delay: Lower and Upper Bounds

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Two Periods of Delay

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Three Periods of Delay

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Consider the Bargaining Game \mathcal{B} with no deadlines. For each finite time period n^{*} and each $x^* \in [\underline{x}_{n^*}, \overline{x}_{n^*}]$, there exists some $(\mathcal{B}, \mathcal{T})$ and a state $(s_1^*, t_1^*, s_2^*, t_2^*)$ thereof, so that

there is forward induction reasoning under no on path strategic uncertainty at (s₁^{*}, t₁^{*}, s₂^{*}, t₂^{*}); and

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2 (s_1^*, s_2^*) induces the outcome $(x^*, 1 - x^*, n^*)$.

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A Mecha	nism for D	Delay		
Introduction	Set-Up	Necessity	Sufficiency	The Nature of Strategic Uncertainty
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A Mecha	nism for E	Delay		

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Introduction	Set-Up	Necessity	Sufficiency	The Nature of Strategic Uncertainty
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A Mecha	nism for E	Delay		

• Each Bargainer thinks will agree on a $(x^* : 1 - x^*)$ split in n^*

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A Mecha	nism for E	Delay		

- Each Bargainer thinks will agree on a $(x^* : 1 x^*)$ split in n^*
 - Each Bargainer thinks other Bargain thinks will agree on a $(x^*: 1 x^*)$ split in n^*

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• There is no "better offer" to be made earlier

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A Mechar	nism for D	Delav		

- Each Bargainer thinks will agree on a $(x^*: 1 x^*)$ split in n^*
 - Each Bargainer thinks other Bargain thinks will agree on a $(x^*: 1 x^*)$ split in n^*
- There is no "better offer" to be made earlier
- Uncertainty about how other Bargainer will react to the unexpected

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A Mecha	nism for E	Delay		

- Each Bargainer thinks will agree on a $(x^*: 1 x^*)$ split in n^*
 - Each Bargainer thinks other Bargain thinks will agree on a $(x^*: 1 x^*)$ split in n^*
- There is no "better offer" to be made earlier
- Uncertainty about how other Bargainer will react to the unexpected

• Other Bargainer will become more optimistic about future prospects

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• Make higher demands

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- Each Bargainer thinks will agree on a $(x^*: 1 x^*)$ split in n^*
 - Each Bargainer thinks other Bargain thinks will agree on a $(x^*: 1 x^*)$ split in n^*
- There is no "better offer" to be made earlier
- Uncertainty about how other Bargainer will react to the unexpected

- Other Bargainer will become more optimistic about future prospects
- Make higher demands

Mechanism Consistent with Forward Induction Reasoning?

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- Each Bargainer thinks will agree on a $(x^*: 1 x^*)$ split in n^*
 - Each Bargainer thinks other Bargain thinks will agree on a $(x^*: 1 x^*)$ split in n^*
- There is no "better offer" to be made earlier
- Uncertainty about how other Bargainer will react to the unexpected
 - Other Bargainer will, by necessity, hypothesize that I am irrational
 - Other Bargainer will become more optimistic about future prospects
 - Make higher demands

Mechanism Consistent with Forward Induction Reasoning?

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A Mechar	nism for E	Delay		

- Each Bargainer thinks will agree on a $(x^*: 1 x^*)$ split in n^*
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- There is no "better offer" to be made earlier
- Uncertainty about how other Bargainer will react to the unexpected
 - Other Bargainer will, by necessity, hypothesize that I am irrational ??
 - Other Bargainer will become more optimistic about future prospects
 - Make higher demands

Mechanism Consistent with Forward Induction Reasoning?

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An Illustration: Three Period Deadline

An Illustration: Three Period Deadline

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An Illustration: Three Period Deadline

Can Construct a Type Structure

- State: $(s_1^*, t_1^*, s_2^*, t_2^*)$
- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: $(\delta, 1 \delta, 1)$

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An Illustration: Three Period Deadline

Can Construct a Type Structure

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- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: $(\delta, 1 \delta, 1)$

An Illustration: Three Period Deadline

Can Construct a Type Structure

- State: ($s_1^*, t_1^*, s_2^*, t_2^*$)
- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: $(\delta, 1 \delta, 1)$

Strategy s₁^{*}:

• Proposer: Offer to take the Full Pie

An Illustration: Three Period Deadline

Can Construct a Type Structure

- State: ($s_1^*, t_1^*, s_2^*, t_2^*$)
- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: (δ, 1 − δ, 1)

Strategy s₁^{*}:

- Proposer: Offer to take the Full Pie
- Responder and Offered (1 x, x, 1): A if and only if $1 x \ge \delta$

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An Illustration: Three Period Deadline

Can Construct a Type Structure

- State: ($s_1^*, t_1^*, s_2^*, t_2^*$)
- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: $(\delta, 1 \delta, 1)$

Strategy s₁^{*}:

- Proposer: Offer to take the Full Pie
- Responder and Offered (1 x, x, 1): A if and only if $1 x \ge \delta$

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An Illustration: Three Period Deadline

Can Construct a Type Structure

- State: ($s_1^*, t_1^*, s_2^*, t_2^*$)
- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: $(\delta, 1 \delta, 1)$

Strategy s₁^{*}:

- Proposer: Offer to take the Full Pie
- Responder and Offered (1 x, x, 1): A if and only if $1 x \ge \delta$

Strategy s₂^{*}:

• Initial Offer (x, 1 - x, 0): A if and only if $1 - x \ge \delta$.

An Illustration: Three Period Deadline

Can Construct a Type Structure

- State: ($s_1^*, t_1^*, s_2^*, t_2^*$)
- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: $(\delta, 1 \delta, 1)$

Strategy s₁^{*}:

- Proposer: Offer to take the Full Pie
- Responder and Offered (1 x, x, 1): A if and only if $1 x \ge \delta$

- Initial Offer (x, 1 x, 0): A if and only if $1 x \ge \delta$.
- Proposer:

An Illustration: Three Period Deadline

Can Construct a Type Structure

- State: ($s_1^*, t_1^*, s_2^*, t_2^*$)
- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: $(\delta, 1 \delta, 1)$

Strategy s₁^{*}:

- Proposer: Offer to take the Full Pie
- Responder and Offered (1 x, x, 1): A if and only if $1 x \ge \delta$

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- Initial Offer (x, 1 x, 0): A if and only if $1 x \ge \delta$.
- Proposer:
 - Initially Offered (1,0,0): Offer $(\delta, 1-\delta, 1)$

An Illustration: Three Period Deadline

Can Construct a Type Structure

- State: $(s_1^*, t_1^*, s_2^*, t_2^*)$
- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: $(\delta, 1 \delta, 1)$

Strategy s₁^{*}:

- Proposer: Offer to take the Full Pie
- Responder and Offered (1 x, x, 1): A if and only if $1 x \ge \delta$

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- Initial Offer (x, 1 x, 0): A if and only if $1 x \ge \delta$.
- Proposer:
 - Initially Offered (1,0,0): Offer $(\delta, 1-\delta, 1)$
 - Otherwise: Offer (0, 1, 1)

An Illustration: Three Period Deadline

Can Construct a Type Structure

- State: ($s_1^*, t_1^*, s_2^*, t_2^*$)
- FI Reasoning under No On Path Strategic Uncertainty
- Outcome: $(\delta, 1 \delta, 1)$

Strategy s₁^{*}:

- Proposer: Offer to take the Full Pie
- Responder and Offered (1 x, x, 1): A if and only if $1 x \ge \delta$

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- Initial Offer (x, 1 x, 0): A if and only if $1 x \ge \delta$.
- Proposer:
 - Initially Offered (1,0,0): Offer $(\delta, 1-\delta, 1)$
 - Otherwise: Offer (0, 1, 1)
- Third Period Offer (z, 1 z, 2): A

A First (and Futile) Attempt

Type Sets: $\{t_1^*\}$ and $\{t_2^*\}$



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A First (and Futile) Attempt

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Belief of t_1^* :

• At Information Set Allowed by s_2^* : Probability 1 to (s_2^*, t_2^*)

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- At Information Set Allowed by s_2^* : Probability 1 to (s_2^*, t_2^*)
- At Information Set *h* Precluded by s_1^* : Probability 1 to $(s_2^{h,*}, t_2^*)$

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A First (and Futile) Attempt: Rationality

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Strategy s₂^{*}:

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Rational Strategy Type Pair: (s_2^*, t_2^*)

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A First (and Futile) Attempt: Rationality

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Rational Strategy Type Pair:

- (s_1^*, t_1^*)
- But second rational strategy-type pair

A First (and Futile) Attempt: Rationality

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Rational Strategy Type Pair:

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• But second rational strategy-type pair

Implication for 2: Does not Strongly Believe 1 is Rational!



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Strategy s₂^{*}:

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Type t_1^* 's Belief

• Now: Strictly Prefer $(\delta, 1 - \delta, 1)$ over (1, 0, 2)



- Initial Offer (x, 1 x, 0): A if and only if $1 x \ge \delta$.
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- Now: Strictly Prefer $(\delta, 1 \delta, 1)$ over (1, 0, 2)
- Now t_2^* Does Strongly Believe R_1^1

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The Problem and Solution

Strategy s^{*}₂:

- Initial Offer (x, 1 x, 0): A if and only if $1 x > \delta$.
- Proposer:
 - Initially Offered (1,0,0): Offer $(\delta, 1-\delta, 1)$
 - Otherwise: Offer (0, 1, 1)
- Third Period Offer (z, 1 z, 2): A if 1 z > 0

- Now: Strictly Prefer $(\delta, 1 \delta, 1)$ over (1, 0, 2)
- Now t_2^* Does Strongly Believe R_1^1
- But: Conditional on Third Period Information Set Being Reached

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Strategy s^{*}:

- Initial Offer (x, 1 x, 0): A if and only if $1 x \ge \delta$.
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- Now t_2^* Does Strongly Believe R_1^1
- But: Conditional on Third Period Information Set Being Reached
 - No Best Response for t_2^*

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- Now t_2^* Does Strongly Believe R_1^1
- But: Conditional on Third Period Information Set Being Reached
 - No Best Response for t_2^*
- If Third Period Information Set is Reached:

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The Problem and Solution

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- Initial Offer (x, 1 x, 0): A if and only if $1 x \ge \delta$.
- Proposer:
 - Initially Offered (1,0,0): Offer $(\delta,1-\delta,1)$
 - Otherwise: Offer (0, 1, 1)
- Third Period Offer (z, 1-z, 2): A if 1-z > 0

- Now: Strictly Prefer $(\delta, 1 \delta, 1)$ over (1, 0, 2)
- Now t_2^* Does Strongly Believe R_1^1
- But: Conditional on Third Period Information Set Being Reached
 - No Best Response for t_2^*
- If Third Period Information Set is Reached:
 - Believe 2 Accepts any Offer



- Initial Offer (x, 1 x, 0): A if and only if $1 x \ge \delta$.
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- Now: Strictly Prefer $(\delta, 1 \delta, 1)$ over (1, 0, 2)
- Now t_2^* Does Strongly Believe R_1^1
- But: Conditional on Third Period Information Set Being Reached
 - No Best Response for t_2^*
- If Third Period Information Set is Reached:
 - Believe 2 Accepts any Offer
 - Can Revise Beliefs: 2 Must be Irrational



- Initial Offer (x, 1 x, 0): A if and only if $1 x \ge \delta$.
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 - Initially Offered (1,0,0): Offer $(\delta, 1-\delta, 1)$
 - Otherwise: Offer (0,1,1)
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Belief of t_2^* :

• If 1 initially Offers (1, 0, 0): Probability 1 to (s_1^*, t_1^*)

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- Otherwise: Probability 1 to
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Rational Strategy Type Pair: (s_2^*, t_2^*)



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Rational Strategy Type Pair: (s_2^*, t_2^*) and ...

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Question

Is the assumption of No on Path Strategic Uncertainty Restrictive?

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Is the assumption of No on Path Strategic Uncertainty Restrictive?

Lessons from Finite Games: Battigalli and Friedenberg (2012)

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• Perfect Information Games satisfying TDI

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Delay and On Path Strategic Uncertainty

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Is the assumption of No on Path Strategic Uncertainty Restrictive?

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- Perfect Information Games satisfying TDI
- Forward Induction Reasoning Rules out On Path Strategic Uncertainty

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Delay and On Path Strategic Uncertainty

Question

Is the assumption of No on Path Strategic Uncertainty Restrictive?

Lessons from Finite Games: Battigalli and Friedenberg (2012)

- Perfect Information Games satisfying TDI
- Forward Induction Reasoning Rules out On Path Strategic Uncertainty

Let \mathcal{B} be the Bargaining Game without a deadline. There exists an epistemic game $(\mathcal{B}, \mathcal{T})$ and an outcome consistent with forward induction reasoning, viz. $(x^*, 1 - x^*, n^*)$, so that $x^* < \underline{x}_{n^*}$.

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The Idea				
Example				

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The Idea				

• Type Set: $T_i = \{t_i^*\}$



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The Idea				

- Type Set: $T_i = \{t_i^*\}$
- *Ex Ante, t*^{*}₁ assigns probability one to RCSBR for Bargainer 2

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- Type Set: $T_i = \{t_i^*\}$
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• *Ex ante*, assigns probability one to $(x^*, 1 - x^*, 2)$:

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$$x^* \geq \underline{x}_2$$

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- *Ex ante*, assigns probability one to $(x^*, 1 x^*, 2)$:
 - $x^* \ge \underline{x}_2$ • If not would profer to offer 1
 - $\bullet~$ If not, would prefer to offer $1-\delta$ in period 0
- If $(y^*, 1 y^*, 4)$ is RCSBR outcome with $x^* < \underline{x}_4$:

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- Type Set: $T_i = \{t_i^*\}$
- *Ex Ante,* t_1^* assigns probability one to RCSBR for Bargainer 2
- *Ex ante*, assigns probability one to $(x^*, 1 x^*, 2)$:
 - $x^* \ge \underline{x}_2$ • If not, would prefer to offer $1 - \delta$ in period 0
- If $(y^*, 1 y^*, 4)$ is RCSBR outcome with $x^* < \underline{x}_4$:
 - Bargainer 2 chooses between $(y^*, 1 y^*, 4)$ and $(x^*, 1 x^*, 2)$

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Bargainer 2 indifferent between these outcomes

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• Bargainer 2 indifferent between these outcomes

Nature of On Path Strategic Uncertainty:

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The Idea				

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- $\bullet~$ If not, would prefer to offer $1-\delta~{\rm in}$ period 0
- If $(y^*, 1 y^*, 4)$ is RCSBR outcome with $x^* < \underline{x}_4$:
 - Bargainer 2 chooses between $(y^*, 1 y^*, 4)$ and $(x^*, 1 x^*, 2)$

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Bargainer 2 indifferent between these outcomes

Nature of On Path Strategic Uncertainty:

• Incorrect Beliefs about how Bargainer 2 Resolves Indifferences

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- Type Set: $T_i = \{t_i^*\}$
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- *Ex ante*, assigns probability one to $(x^*, 1 x^*, 2)$:

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$$x^* \geq \underline{x}_2$$

- ullet If not, would prefer to offer $1-\delta$ in period 0
- If $(y^*, 1 y^*, 4)$ is RCSBR outcome with $x^* < \underline{x}_4$:
 - Bargainer 2 chooses between $(y^*, 1 y^*, 4)$ and $(x^*, 1 x^*, 2)$
 - Bargainer 2 indifferent between these outcomes

Nature of On Path Strategic Uncertainty:

- Incorrect Beliefs about how Bargainer 2 Resolves Indifferences
- Bargainer 1 not Indifferent

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- $\bullet~$ If not, would prefer to offer 1 $-~\delta$ in period 0
- If $(y^*, 1 y^*, 4)$ is RCSBR outcome with $x^* < \underline{x}_4$:
 - Bargainer 2 chooses between $(y^*, 1 y^*, 4)$ and $(x^*, 1 x^*, 2)$

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Bargainer 2 indifferent between these outcomes

Nature of On Path Strategic Uncertainty:

- Incorrect Beliefs about how Bargainer 2 Resolves Indifferences
- Bargainer 1 not Indifferent
- Failure of TDI

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 Predictions of On-Path Strategic Uncertainty and Delay

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Causes of On-Path Strategic Uncertainty:




Predictions of On-Path Strategic Uncertainty and Delay

Causes of On-Path Strategic Uncertainty:

Uncertainty about "how a given type plays"



Causes of On-Path Strategic Uncertainty:

- Uncertainty about "how a given type plays"
 - Under Rationality: Uncertainty about how resolve indifferences

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Causes of On-Path Strategic Uncertainty:

- Uncertainty about "how a given type plays"
 - Under Rationality: Uncertainty about how resolve indifferences

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Output in the second second



Causes of On-Path Strategic Uncertainty:

- Uncertainty about "how a given type plays"
 - Under Rationality: Uncertainty about how resolve indifferences
- Output about first-order beliefs

Proposition

Fix some $(\mathcal{B}, \mathcal{T})$ so that there are a finite number of terminal nodes consistent with forward induction reasoning. Then, there must be states $(s_i, t_i, s_{-i}, t_{-i})$ and $(r_i, t_i, s_{-i}, t_{-i})$



Causes of On-Path Strategic Uncertainty:

- Uncertainty about "how a given type plays"
 - Under Rationality: Uncertainty about how resolve indifferences
- Output about first-order beliefs

Proposition

Fix some $(\mathcal{B}, \mathcal{T})$ so that there are a finite number of terminal nodes consistent with forward induction reasoning. Then, there must be states $(s_i, t_i, s_{-i}, t_{-i})$ and $(r_i, t_i, s_{-i}, t_{-i})$

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- Bargainer i is the only player indifferent between z and z'.

- Battigalli, P. and A. Friedenberg. 2012. "Forward Induction Reasoning Revisited." *Theoretical Economics* 7:57–98.
- Battigalli, P. and M. Siniscalchi. 2002. "Strong Belief and Forward Induction Reasoning." *Journal of Economic Theory* 106(2):356–391.
- Kohlberg, E. 1981. "Some Problems with the Concept of Perfect Equilibrium." *Rapp. Rep. NBER Conf. Theory Gen. Econ. Equilibr. K. Dunz N. Singh, Univ. Calif. Berkeley*.

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