Bargaining Under Strategic Uncertainty

Amanda Friedenberg

Extremely Extremely Preliminary

Bargaining

Bargaining

Key Feature of Many Applications:

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- Employment Contracts
- Trials and Arbitration
- Sovereign Debt
- War
- Legislative Bargaining
- etc.

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- Employment Contracts
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- etc.

Important Behavioral Feature:

- Failure to Reach Immediate Agreement

A Source of Bargaining Impasse

Strategic Uncertainty

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Strategic Uncertainty

Two Concerns:

A Source of Bargaining Impasse

Strategic Uncertainty

Two Concerns:

1. Too Many Predictions?

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Strategic Uncertainty

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1. Too Many Predictions?
   - Sophisticated Reasoning about Strategic Uncertainty?

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Strategic Uncertainty

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1. Too Many Predictions?
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    - Limit predictions to rule out impasse?

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Forward Induction Reasoning

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Forward Induction Reasoning: Kohlberg, 1981

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- Rationalize Past Behavior when Possible

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  - Rationality and Common Strong Belief of Rationality

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  - Property of (s_i, b_i^1)

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  - Assign Probability 1 to event "Rational" when Possible
- And so on.

Lessons from Finite Games

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Belief Dependent Concept

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Formally:

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- Type Structure: Hierarchies of Beliefs about the Play of the Game

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- Behavioral Predictions can change with Type Structure
- In Particular:
  - "Rich" Type Structure: Extensive-Form Rationalizability
    - Battigalli and Siniscalchi, 2002
  - "Small" Type Structure: Disjoint Prediction

Lessons from Finite Games

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Conceptually:

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- How Does a Player Update His Hypothesis when Surprised?

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  - (a) Hypothesis that other player is rational vs.
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  - Limit the ability to Give up on (b)

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- Tension between Giving up on:
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- Forward Induction Reasoning: Give up on (b)
- Small Type Structures:
  - Limit the ability to Give up on (b)
- What Small Type Structures are Meant to Capture
  - Restrictions on Players' Beliefs
  - Game Described as Part of a Bigger Context

Sophisticated Reasoning about Strategic Uncertainty

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A. Restriction on Players' Beliefs

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  - About Terminal Node

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  - Applications

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- Co-player satisfies No On Path Strategic Uncertainty

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B. Forward Induction Reasoning

- Co-player satisfies No On Path Strategic Uncertainty
- Limits ability to rationalize co-player's past behavior

Behavioral Implications

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Main Theorem

Characterize the outcomes consistent with Forward Induction Reasoning under No On Path Strategic Uncertainty.

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   - Can have impasse

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   - Can have impasse
   - Depends on the deadline (if there is any)
   - Depends on Bargainers' patience

Bargaining Game: $\mathcal{B}$

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Timeline

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0P Bargainer 1 Offers: $x \in [0, 1]$

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- Quitting Period: N
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2P ...

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- Quitting Period: N
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  - N Infinite: No Deadline
- Payoffs: Share of z in period n gives $\delta^n z$

Modeling Strategic Uncertainty

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What we will Need:

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b_i^1 Bargainer *i*'s beliefs about how other plays

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- Bargainer *i* may begin the game with one hypothesis

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- etc.

What we will Really Need:

- Bargainer i may begin the game with one hypothesis
- May be forced to revise beliefs
  - If other Bargainer plays differently
- Hierarchies of Conditional Beliefs about the Play

Type Structures

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Ingredients of a Type Structure

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For each Player i:

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For each Player i:

1. Type Set: T_i

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For each Player i:

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   - Map type t_i to belief on $S_{-i} \times T_{-i}$

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    - To system of beliefs on $S_{-i} \times T_{-i}$

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     - One belief for each information set

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        - One belief for each information set
        - Satisfy rules of conditional probability if possible

How to Think of the Objects on the Table

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Description

Epistemic Game: $(\mathcal{B}, \mathcal{T})$

1. Bargaining Game
2. Type Structure

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Steps to Formalization:

Forward Induction Reasoning

Forward Induction

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2. Strong Belief: "Thinking"

Forward Induction Reasoning

Forward Induction

Rationalize Past Behavior When Possible

Steps to Formalization:

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   - Assign probability 1 to E_{-i}, if $E_{-i} \cap [S_{-i}(h) \times T_{-i}] \neq \emptyset$

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Steps to Formalization:

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Steps to Formalization:

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   - Assign probability 1 to R_{-i}^1, if $R_{-i}^1 \cap [S_{-i}(h) \times T_{-i}] \neq \emptyset$

No On Path Strategic Uncertainty

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Say there is **no on path strategic uncertainty at a state** (s_1, t_1, s_2, t_2) if, for each information set along the path of play induced by (s_1, s_2), t_1 (resp. t_2) assigns probability 1 to reaching the terminal node, viz. z^*, induced by (s_1, s_2).

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Rephrase:

- Event $\mathbb{Z}_{-i}[s_1, s_2]$:
  - Event the terminal node associated with (s_1, s_2), viz. z^*, is reached, when s_i is played
- Each t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$

Epistemic Conditions

Epistemic Conditions

Level 1

- Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

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Level 2

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$
- t_i strongly believes R_{-i}^1

Epistemic Conditions

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- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$

Epistemic Conditions

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Level 2

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

Epistemic Conditions

Level 1

- Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

Level 3

Epistemic Conditions

Level 1

- Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

Level 3

- Survives Level 2: $(s_1, t_1, s_2, t_2) \in R_1^2 \times R_2^2$

Epistemic Conditions

Level 1

- Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2: $R_1^2 \times R_2^2$

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

Level 3

- Survives Level 2: $(s_1, t_1, s_2, t_2) \in R_1^2 \times R_2^2$
- t_i strongly believes R_{-i}^2

Epistemic Conditions

Level 1

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Level 2: $R_1^2 \times R_2^2$

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R_{-i}^1 Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

Level 3

- Survives Level 2: $(s_1, t_1, s_2, t_2) \in R_1^2 \times R_2^2$
- t_i strongly believes R_{-i}^2

Level 4 ...

Epistemic Conditions

Level 1

- Rationality: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$

Level 2: $R_1^2 \times R_2^2$

- Survives Level 1: $(s_1, t_1, s_2, t_2) \in R_1^1 \times R_2^1$ Rationality
- t_i strongly believes R^1_{-i} Strong Belief of Rationality
- t_i strongly believes $\mathbb{Z}_{-i}[s_1, s_2]$ No On Path Strategic Uncertainty

Level 3

- Survives Level 2: $(s_1, t_1, s_2, t_2) \in R_1^2 \times R_2^2$
- t_i strongly believes R^2_{-i}

Level 4 . . .

Forward Induction Reasoning Under No On Path Strategic Uncertainty

Bounds on Delay

Bounds on Delay

Bounds on outcomes will come from two levels of reasoning:

- Rationality
- Strong Belief of Rationality
- No On Path Strategic Uncertainty

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Examples:

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Examples:

1. Two Period Example: No Delay

Bounds on Delay

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Examples:

1. Two Period Example: No Delay
2. Three Period Example: If Delay then $(\delta, 1-\delta, 1)$

Bounds on Delay

Bounds on outcomes will come from two levels of reasoning:

- Rationality
- Strong Belief of Rationality
- No On Path Strategic Uncertainty

Examples:

1. Two Period Example: No Delay
2. Three Period Example: If Delay then $(\delta, 1-\delta, 1)$
   - Only Happen if δ sufficiently large

The Two Period Deadline

The Two Period Deadline

Suppose Delay

The Two Period Deadline

Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$

The Two Period Deadline

Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$
- Along Path: 2 Proposes

The Two Period Deadline

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- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$
- Along Path: 2 Proposes
- t_2^* Strongly Believes 1 is Rational

The Two Period Deadline

Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1-x^*, 1)$
- Along Path: 2 Proposes
- t_2^* Strongly Believes 1 is Rational
    - When Propose: Continues to believe 1 is Rational

The Two Period Deadline

Suppose Delay

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- (s_2^*, t_2^*) Rational and Strongly Believes Rational:
  - 2 offers to take the full pie and expects 1 to Accept

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- Strong Belief of Rationality
  - t_1^* begins the game believing: 2 Accepts any $x < 1 - \delta$ upfront

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- No On Path Strategic Uncertainty:
  - t_1^* begins the game believing: $(0, 1, 1)$
- Strong Belief of Rationality
  - t_1^* begins the game believing: 2 Accepts any $x < 1 - \delta$ upfront
- 1 would strictly prefer to offer some $x < 1 - \delta$ split upfront

Three Period Deadline

Three Period Deadline

Suppose Delay

Three Period Deadline

Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$

Three Period Deadline

Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$
- Look at Path Induced by State $(s_1^*, t_1^*, s_2^*, t_2^*)$

Three Period Deadline

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- When 2 Proposes $1-x^*$:

Three Period Deadline

Suppose Delay

- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$
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- When 2 Proposes $1 - x^*$:
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  - So: $1 - x^* \geq 1 - \delta$ or $\delta \geq x^*$

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- When 1 Accepts $1 - \delta$

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  - Believes 1 will Accept any Offer $y < 1 - \delta$
  - So: $1 - x^* \geq 1 - \delta$ or $\delta \geq x^*$
- When 1 Accepts $1 - \delta$
  - Continues to Believe 2 is Rational

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  - Believes 2 will Accept any third-period Offer $z < 1$

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Three Period Deadline Revisited

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Three Period Deadline Revisited

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- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1-x^*, 1)$
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- **Upper Bound:** δ

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- **Lower Bound Bound:**

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- So: $\delta x^* \geq 1 - \delta$ or $x^* \geq \frac{1-\delta}{\delta}$

- **Upper Bound:** δ
- **Lower Bound Bound:**
  - No Incentive to Delay till Deadline: δ
  - No Incentive to Settle Upfront: $\frac{1-\delta}{\delta}$

Three Period Deadline Revisited

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- Outcome $(s_1^*, t_1^*, s_2^*, t_2^*)$: $(x^*, 1 - x^*, 1)$
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Three Period Deadline Revisited

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  - Believes 2 will Accept any Offer $y < 1-\delta$
- So: $\delta x^* \geq 1-\delta$ or $x^* \geq \frac{1-\delta}{\delta}$

- **Upper Bound:** δ
- **Lower Bound Bound:**
  - No Incentive to Delay till Deadline: δ
  - No Incentive to Settle Upfront: $\frac{1-\delta}{\delta}$
- Delay only if sufficiently patient

No Deadline: Necessity

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No Deadline: Necessity

Theorem

Fix some epistemic game $(\mathcal{B}, \mathcal{T})$ *with no deadline. Suppose that, at* $(s_1^*, t_1^*, s_2^*, t_2^*)$

- *each player is rational*
- *each player strongly believes the other player is rational, and*
- *there is no on path strategic uncertainty.*

Then, (s_1^*, s_2^*) *induces an outcome* $(x^*, 1 - x^*, n^*)$ *with* $x^* \in [\underline{x}_{n^*}, \overline{x}_{n^*}]$.

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- *each player is rational*
- *each player strongly believes the other player is rational, and*
- *there is no on path strategic uncertainty.*

Then, (s_1^*, s_2^*) induces an outcome $(x^*, 1-x^*, n^*)$ with $x^* \in [\underline{x}_{n^*}, \overline{x}_{n^*}]$.

$$\underline{x}_{n^*} = \frac{1-\delta}{\delta^{n^*}} \quad \text{and} \quad \overline{x}_{n^*} = \begin{cases} 1 - \frac{\delta(1-\delta)}{\delta^{n^*}} & \text{if } n^* \geq 1 \\ 1 & \text{if } n^* = 0. \end{cases}$$

Illustration of Possible Outcomes: No Deadline

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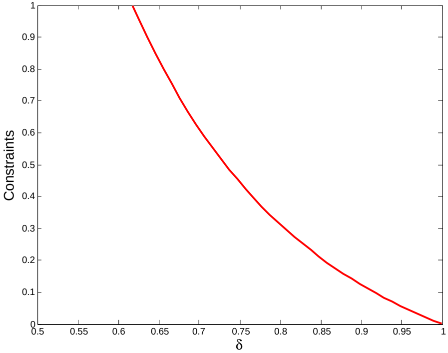
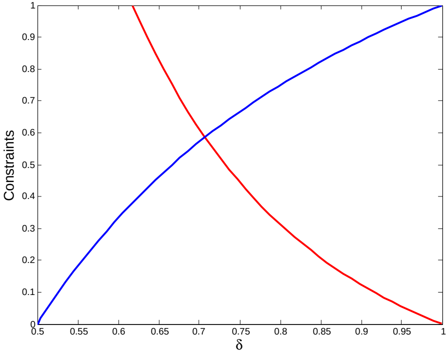


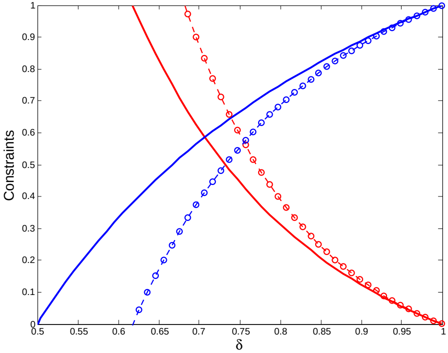
One Period of Delay: Lower Bound

Illustration of Possible Outcomes: No Deadline



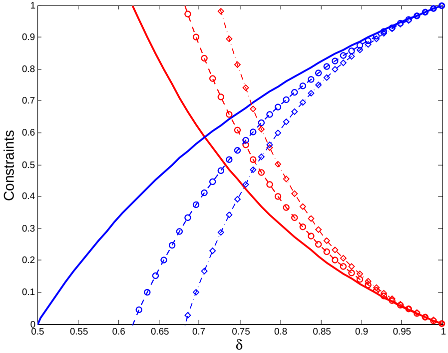
One Period of Delay: Lower and Upper Bounds

Illustration of Possible Outcomes: No Deadline



Two Periods of Delay

Illustration of Possible Outcomes: No Deadline



Three Periods of Delay

Sufficiency: No Deadline

Theorem

Consider the Bargaining Game $\mathcal{B}$ with no deadlines. For each finite time period n^* and each $x^* \in [\underline{x}_{n^*}, \overline{x}_{n^*}]$, there exists some $(\mathcal{B}, \mathcal{T})$ and a state $(s_1^*, t_1^*, s_2^*, t_2^*)$ thereof, so that

1. *there is forward induction reasoning under no on path strategic uncertainty at $(s_1^*, t_1^*, s_2^*, t_2^*)$; and*
2. *(s_1^*, s_2^*) induces the outcome $(x^*, 1 - x^*, n^*)$.*

A Mechanism for Delay

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- Each Bargainer thinks will agree on a $(x^* : 1 - x^*)$ split in n^*

A Mechanism for Delay

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- Each Bargainer thinks will agree on a $(x^* : 1 - x^*)$ split in n^*
  - Each Bargainer thinks other Bargain thinks will agree on a $(x^* : 1 - x^*)$ split in n^*

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- Each Bargainer thinks will agree on a $(x^* : 1 - x^*)$ split in n^*
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- Uncertainty about how other Bargainer will react to the unexpected
    - Other Bargainer will become more optimistic about future prospects
    - Make higher demands

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Mechanism Consistent with Forward Induction Reasoning?

A Mechanism for Delay

A Mechanism

- Each Bargainer thinks will agree on a $(x^* : 1 - x^*)$ split in n^*
  - Each Bargainer thinks other Bargain thinks will agree on a $(x^* : 1 - x^*)$ split in n^*
- There is no "better offer" to be made earlier
- Uncertainty about how other Bargainer will react to the unexpected
  - Other Bargainer will, by necessity, hypothesize that I am irrational
  - Other Bargainer will become more optimistic about future prospects
  - Make higher demands

Mechanism Consistent with Forward Induction Reasoning?

A Mechanism for Delay

A Mechanism

- Each Bargainer thinks will agree on a $(x^* : 1 - x^*)$ split in n^*
  - Each Bargainer thinks other Bargain thinks will agree on a $(x^* : 1 - x^*)$ split in n^*
- There is no "better offer" to be made earlier
- Uncertainty about how other Bargainer will react to the unexpected
  - Other Bargainer will, by necessity, hypothesize that I am irrational ??
  - Other Bargainer will become more optimistic about future prospects
  - Make higher demands

Mechanism Consistent with Forward Induction Reasoning?

An Illustration: Three Period Deadline

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Can Construct a Type Structure

An Illustration: Three Period Deadline

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- State: $(s_1^*, t_1^*, s_2^*, t_2^*)$
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- Outcome: $(\delta, 1 - \delta, 1)$

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A First (and Futile) Attempt

Type Sets: $\{t_1^*\}$ and $\{t_2^*\}$

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- At Information Set Allowed by s_2^*: Probability 1 to (s_2^*, t_2^*)

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A First (and Futile) Attempt: Rationality

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Implication for 2: Does not Strongly Believe 1 is Rational!

The Problem and Solution

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Type t_1^*'s Belief

- Now: Strictly Prefer $(\delta, 1-\delta, 1)$ over $(1, 0, 2)$

The Problem and Solution

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- Now t_2^* Does Strongly Believe R_1^1

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- Now: Strictly Prefer $(\delta, 1-\delta, 1)$ over $(1, 0, 2)$
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- But: Conditional on Third Period Information Set Being Reached

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  - Can Revise Beliefs: 2 Must be Irrational

A Second Problem

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Rational Strategy Type Pair: (s_2^*, t_2^*)

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Delay and On Path Strategic Uncertainty

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Question

Is the assumption of No on Path Strategic Uncertainty Restrictive?

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- Perfect Information Games satisfying TDI

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- Perfect Information Games satisfying TDI
- Forward Induction Reasoning Rules out On Path Strategic Uncertainty

Delay and On Path Strategic Uncertainty

Question

Is the assumption of No on Path Strategic Uncertainty Restrictive?

Lessons from Finite Games: Battigalli and Friedenberg (2012)

- Perfect Information Games satisfying TDI
- Forward Induction Reasoning Rules out On Path Strategic Uncertainty

Let $\mathcal{B}$ be the Bargaining Game without a deadline. There exists an epistemic game $(\mathcal{B}, \mathcal{T})$ and an outcome consistent with forward induction reasoning, viz. $(x^*, 1 - x^*, n^*)$, so that $x^* < \underline{x}_{n^*}$.

The Idea

Example

The Idea

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- Type Set: $T_i = \{t_i^*\}$

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- *Ex Ante*, t_1^* assigns probability one to RCSBR for Bargainer 2

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- Type Set: $T_i = \{t_i^*\}$
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    - $x^* \geq \underline{x}_2$

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- Type Set: $T_i = \{t_i^*\}$
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- If $(y^*, 1 - y^*, 4)$ is RCSBR outcome with $x^* < \underline{x}_4$:

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  - Bargainer 2 indifferent between these outcomes

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Nature of On Path Strategic Uncertainty:

The Idea

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- Incorrect Beliefs about how Bargainer 2 Resolves Indifferences

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- Incorrect Beliefs about how Bargainer 2 Resolves Indifferences
- Bargainer 1 not Indifferent

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Nature of On Path Strategic Uncertainty:

- Incorrect Beliefs about how Bargainer 2 Resolves Indifferences
- Bargainer 1 not Indifferent
- Failure of TDI

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4. *Bargainer i is the only player indifferent between z and z'.*

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