The dynamics of lying

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ANTON PIECK

Das Dietmarsische Lügenmärchen

Ich will euch etwas erzählen. Ich sah zwei gebratene Hühner fliegen, flogen schnell und hatten die Bäuche gen Himmel gekehrt, die Rücken nach der Hölle, und ein Amboß und ein Mühlstein schwammen ber den Rhein, fein langsam und leise, und ein Frosch saß und fraß eine Pflugschar zu Pfingsten auf dem Eis. Da waren drei Kerle, wollten einen Hasen fangen, gingen auf Krücken und Stelzen, der eine war taub, der zweite blind, der dritte stumm und der vierte konnte keinen Fuß rühren. Wollt ihr wissen, wie das geschah? Der Blinde, der sah zuerst den Hasen ber Feld traben, der Stumme rief dem Lahmen zu, und der Lahme faßte ihn beim Kragen. Etliche, die wollten zu Land segeln und spannten die Segel im Wind und schifften ber große Äcker hin: da segelten sie ber einen hohen Berg, da mußten sie elendig ersaufen. Ein Krebs jagte einen Hasen in die Flucht, und hoch auf dem Dach lag eine Kuh, die war hinaufgestiegen. In dem Lande sind die Fliegen so groß als hier die Ziegen. Mache das Fenster auf, damit die Lügen hinausfliegen.

Gebrüder Grimm

The Ditmarsch Tale of Wonders

I will tell you something. I saw two roasted fowls flying; they flew quickly and had their breasts turned to Heaven and their backs to Hell; and an anvil and a mill-stone swam across the Rhine prettily, slowly, and gently; and a frog sat on the ice at Whitsuntide and ate a ploughshare.

. . .

Open the window that the lies may fly out.

Jacob Ludwig Grimm and Wilhelm Carl Grimm, Fairy Tales

Lying and truth telling

In the Grimm Brothers fairy tale it is clear that the speaker lies.

Can you lie without the listener noticing?

What are the informative consequences of lying?

Lying and truth telling

In the Grimm Brothers fairy tale it is clear that the speaker lies.
Can you lie without the listener noticing?
What are the informative consequences of lying?

What are the informative consequences of telling the truth?
Let us recall public announcement logic.

Consecutive numbers

Anne and Bill are each going to be told a natural number. Their numbers will be one apart. The numbers are now being whispered in their respective ears. They are aware of this scenario. Suppose Anne is told 2 and Bill is told 3.
The following truthful conversation between Anne and Bill now takes place:

- *Anne: "I do not know your number."*
- *Bill: "I do not know your number."*
- *Anne: "I know your number."*
- *Bill: "I know your number."*

Explain why is this possible.

Consecutive numbers — representing uncertainties

$$(1{,}0) - a - (1{,}2) - b - (3{,}2) - a - (3{,}4) \text{———} \cdots$$

$$(0{,}1) - b - (2{,}1) - a - \underline{(2{,}3)} - b - (4{,}3) \text{———} \cdots$$

Consecutive numbers — successive announcements

$$(1,0) - a - (1,2) - b - (3,2) - a - (3,4) \text{———} \cdots$$

$$(0,1) - b - (2,1) - a - \underline{(2,3)} - b - (4,3) \text{———} \cdots$$

- Anne: “I do not know your number.” ??

Consecutive numbers — successive announcements

$$(1,0) \overset{a}{—} (1,2) \overset{b}{—} (3,2) \overset{a}{—} (3,4) \text{———} \cdots$$

$$(0,1) \overset{b}{—} (2,1) \overset{a}{—} \underline{(2,3)} \overset{b}{—} (4,3) \text{———} \cdots$$

- Anne: "I do not know your number." eliminated states

Consecutive numbers — successive announcements

$$(1,0) - a - (1,2) - b - (3,2) - a - (3,4) \text{ ———— } \cdots$$

$$(2,1) - a - \underline{(2,3)} - b - (4,3) \text{ ———— } \cdots$$

- Anne: “I do not know your number.”

Consecutive numbers — successive announcements

$$(1,0) - a - (1,2) - b - (3,2) - a - (3,4) \text{———} \cdots$$

$$(2,1) - a - \underline{(2,3)} - b - (4,3) \text{———} \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number." ??

Consecutive numbers — successive announcements

$$(1,0) - a - (1,2) - b - (3,2) - a - (3,4) \text{———} \cdots$$

$$(2,1) - a - \underline{(2,3)} - b - (4,3) \text{———} \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number." eliminated states

Consecutive numbers — successive announcements

$$(1,2) - b - (3,2) - a - (3,4) \text{------} \cdots$$

$$\underline{(2,3)} - b - (4,3) \text{------} \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number."

Consecutive numbers — successive announcements

$$(1{,}2) - b - (3{,}2) - a - (3{,}4) \text{———} \cdots$$

$$\underline{(2{,}3)} - b - (4{,}3) \text{———} \cdots$$

- Anne: “I do not know your number.”
- Bill: “I do not know your number.”
- Anne: “I know your number.” ??

Consecutive numbers — successive announcements

$$(1,2) - b - (3,2) - a - (3,4) \;\text{———}\; \cdots$$

$$\underline{(2,3)} - b - (4,3) \;\text{———}\; \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number." eliminated states

Consecutive numbers — successive announcements

(1,2)

$\underline{(2,3)}$

- Anne: “I do not know your number.”
- Bill: “I do not know your number.”
- Anne: “I know your number.”

Consecutive numbers — successive announcements

(1,2)	
	(2,3)

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number."
- Bill: "I know your number." ??

Consecutive numbers — successive announcements

(1,2)

(2,3)

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number."
- Bill: "I know your number." already common knowledge

Consecutive numbers — successive announcements

(1,2)

<u>(2,3)</u>

- Anne: “I do not know your number.”
- Bill: “I do not know your number.”
- Anne: “I know your number.”
- Bill: “I know your number.”

Public announcement logic

- Jan Plaza, Logics of public communications, 1989 & 2007

We can model truth-telling in public announcement logic.
We can model lying in versions and extensions of that logic.

Let us start with an example...

Consecutive numbers, with lying

$$(0,1) - b - (2,1) - a - \underline{(2,3)} - b - (4,3) \text{———} \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number."
- Bill: "I know your number."

Consecutive numbers, with lying

$$(0,1) - b - (2,1) - a - \underline{(2,3)} - b - (4,3) \text{———} \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number."
- Bill: "I know your number."
- Anne: "I know your number." Anne is lying

Consecutive numbers, with lying

$$(0,1) - b - (2,1) - a - \underline{(2,3)} - b - (4,3) \text{———} \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number."
- Bill: "I know your number."

- Anne: "I know your number." Anne is lying
- Bill: "You're lying."

Consecutive numbers, with lying

$$(0,1) - b - (2,1) - a - \underline{(2,3)} - b - (4,3) \text{———} \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number."
- Bill: "I know your number."

Consecutive numbers, with lying

$$\boxed{(0{,}1) - b - (2{,}1) - a - \underline{(2{,}3)} - b - (4{,}3) \text{———} \cdots}$$

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number."
- Bill: "I know your number."

- Anne: "I do not know your number."

Consecutive numbers, with lying

$$(0{,}1) - b - (2{,}1) - a - \underline{(2{,}3)} - b - (4{,}3) \text{———} \cdots$$

- Anne: "I do not know your number."
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- Bill: "I know your number."

- Anne: "I do not know your number."
- Bill: "I know your number."

Consecutive numbers, with lying

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- Anne: "I do not know your number."
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- Bill: "I know your number."

- Anne: "I do not know your number."
- Bill: "I know your number." Bill is lying

Consecutive numbers, with lying

$$(0,1) - b - (2,1) - a - \underline{(2,3)} - b - (4,3) \text{———} \cdots$$

- Anne: "I do not know your number."
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- Bill: "I know your number."

- Anne: "I do not know your number."
- Bill: "I know your number." Bill is lying
- Anne: "I know your number."

Consecutive numbers, with lying

$$(0,1) - b - (2,1) - a - \underline{(2,3)} - b - (4,3) ——— \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number."
- Bill: "I know your number."

- Anne: "I do not know your number."
- Bill: "I know your number." Bill is lying
- Anne: "I know your number." Anne is mistaken. Anne *thinks* to know that Bill has 1.

Consecutive numbers, with lying

$$(0{,}1) - b - (2{,}1) - a - \underline{(2{,}3)} - b - (4{,}3) \text{———} \cdots$$

- Anne: "I do not know your number."
- Bill: "I do not know your number."
- Anne: "I know your number."
- Bill: "I know your number."

- Anne: "I do not know your number."
- Bill: "I know your number." Bill is lying
- Anne: "I know your number." Anne is mistaken. Anne *thinks* to know that Bill has 1.
- (Bill: "I know your number." By now, this is true!)

What is a lie?

- You are lying if you say to me that φ (is true), but believe that $\neg\varphi$ (is true).
- The lie is effective if I now believe that φ was true. ('Was', not 'is', for technical reasons.)
- For me to believe your lie that φ, I must consider it possible that φ. (Otherwise I will believe that you're lying!)

Lying by an outside observer

- The single agent is the listener whose beliefs are modelled.
- Lies are announcements made by an outsider (not modelled).
- *First version:* The announcements are always believed.

Ignorance

Let p be the proposition 'Oranges freeze in Sevilla'.
Agent a does not know whether this is true.
This uncertainty can be modelled as follows:

$$\neg p \;—\; a \;—\; \underline{p}$$

After the announcement of p we get:

$$\neg p \;—\; a \;—\; \underline{p} \qquad \Rightarrow\ !p \qquad \underline{p}$$

To model lying, later on, we need a more explicit visualization:

$$\begin{tikzcd} \neg p \arrow[loop left, "a"] \arrow[r, leftrightarrow, "a"] & \underline{p} \arrow[loop right, "a"] \end{tikzcd} \qquad \Rightarrow\ !p \qquad \begin{tikzcd} \underline{p} \arrow[loop right, "a"] \end{tikzcd}$$

A different semantics for announcements

An alternative to the logic of public announcements is the logic of *believed announcements.* The effect of the announcement of φ is that only states where φ is true are accessible for the agents. The announcement may be false. This has been called *manipulative announcement* (update).

$$
\begin{tikzcd}
\neg p \arrow[loop left, "a"] \arrow[r, leftrightarrow, "a"] & \underline{p} \arrow[loop right, "a"]
\end{tikzcd}
\quad \Rightarrow !p \quad
\begin{tikzcd}
\neg p \arrow[r, "a"] & \underline{p} \arrow[loop right, "a"]
\end{tikzcd}
$$

After the announcement, the agent believes that Oranges freeze in Sevilla. It is unrelated to whether this is true.

Alternative semantics for announcement:

- Jelle Gerbrandy, Bisimulations on Planet Kripke, ILLC 1999
- Barteld Kooi, Expressivity (...) via reduction axioms. Journal of Applied Non-Classical Logics 17(2): 231-253, 2007

Lie

In case of a lie that p:

- p is false;
- it is announced that p is true;
- after the announcement, agent a believes that p.

There one execution of manipulative announcement:

$$\begin{tikzcd} \underline{\neg p} \arrow[loop left, "a"] \arrow[r, leftrightarrow, "a"] & p \arrow[loop right, "a"] \end{tikzcd} \quad \Rightarrow \text{¡}p \quad \begin{tikzcd} \underline{\neg p} \arrow[r, "a"] & p \arrow[loop right, "a"] \end{tikzcd}$$

The other execution of manipulative announcement is:

$$\begin{tikzcd} \neg p \arrow[loop left, "a"] \arrow[r, leftrightarrow, "a"] & \underline{p} \arrow[loop right, "a"] \end{tikzcd} \quad \Rightarrow\ !p \quad \begin{tikzcd} \neg p \arrow[r, "a"] & \underline{p} \arrow[loop right, "a"] \end{tikzcd}$$

Principles of public lying

Axioms for truthful public announcement:

$$\begin{array}{rcl} [!\varphi]p & \leftrightarrow & \varphi \rightarrow p \\ [!\varphi]\neg\psi & \leftrightarrow & \varphi \rightarrow \neg[!\varphi]\psi \\ [!\varphi](\psi_1 \wedge \psi_2) & \leftrightarrow & [!\varphi]\psi_1 \wedge [!\varphi]\psi_2 \\ [!\varphi]B_i\psi & \leftrightarrow & \varphi \rightarrow B_i[!\varphi]\psi \end{array}$$

Dual axioms for lying:

$$\begin{array}{rcl} [¡\varphi]p & \leftrightarrow & \neg\varphi \rightarrow p \\ [¡\varphi]\neg\psi & \leftrightarrow & \neg\varphi \rightarrow \neg[¡\varphi]\psi \\ [¡\varphi](\psi_1 \wedge \psi_2) & \leftrightarrow & [¡\varphi]\psi_1 \wedge [¡\varphi]\psi_2 \\ [¡\varphi]B_i\psi & \leftrightarrow & \neg\varphi \rightarrow B_i[!\varphi]\psi \end{array}$$

Combined the principles deliver the familiar axiom:

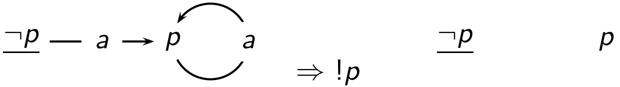
$$[\sharp\varphi]B_i\psi \quad \leftrightarrow \quad B_i(\varphi \rightarrow [\sharp\varphi]\psi)$$

Principles of public lying

$$[¡\varphi]B_i\psi \quad\leftrightarrow\quad \neg\varphi \rightarrow B_i[!\varphi]\psi$$

After the lie that φ agent i believes that ψ, iff,
on condition that φ is false, agent i believes that ψ after truthful announcement that φ.

A problem with lying (and with truthful announcements):
agents going crazy (empty access / believing everything).

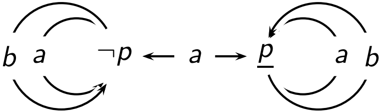


- Hans van Ditmarsch, Jan van Eijck, Yanjing Wang, Floor Sietsma. *On the logic of lying*, LNCS 7010, pp. 41-72, 2012.

Agent-to-agent lying

In public lying it is implicit that the speaker believes that the announcement is false. We can make this explicit in multi-agent epistemic logic.

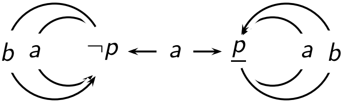
Consider the information state where a does not know whether p, b knows whether p, and p is true.



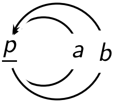
Oranges freeze in Sevilla.
Bill (b) knows whether this is true.
Anne (a) is ignorant.
(And this is common knowledge.)

More agents

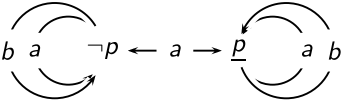
Clearly, a public announcement is not a lie from b to a.



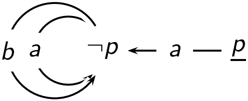
$\Rightarrow\ !p$



A public lie is also not a lie from b to a.

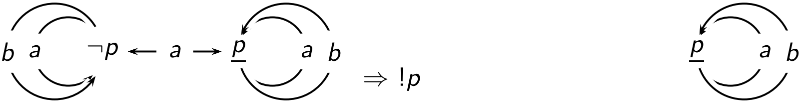


$\Rightarrow\ ¡\neg p$

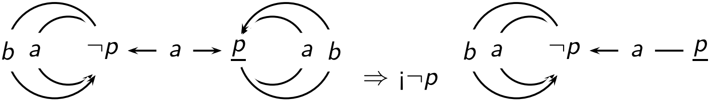


More agents

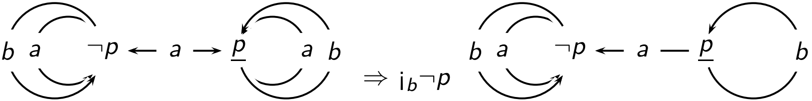
Clearly, a public announcement is not a lie from b to a.



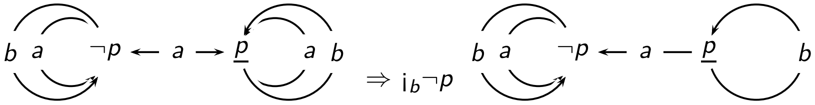
A public lie is also not a lie from b to a.



Instead, a lie from b to a should have the following effect:



Agent b lies to a that $\neg p$



After this successful lie we have that:

- b still believes that p;
- a believes that $\neg p$;
- a believes that a and b have common belief of $\neg p$.

Agent b is lying or telling the truth to agent a that φ

- States where b believes φ remain accessible to b.
- States where b believes $\neg\varphi$ remain accessible to b.
- States where b believes φ remain accessible to a.
- States where b believes $\neg\varphi$ are no longer accessible to a.

Preconditions of agent announcements (by b) that φ

- Truthful agent announcement: $B_b\varphi$
- Lying agent announcement: $B_b\neg\varphi$
- Bluffing agent announcement: $\neg(B_b\varphi \vee B_b\neg\varphi)$

Principles for b lying to a that φ

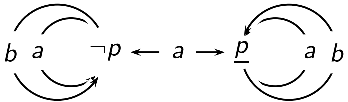
$$
\begin{array}{rcl}
[¡_b\varphi]B_a\psi & \leftrightarrow & B_b\neg\varphi \rightarrow B_a[!_b\varphi]\psi \\
[¡_b\varphi]B_b\psi & \leftrightarrow & B_b\neg\varphi \rightarrow B_b[¡_b\varphi]\psi
\end{array}
$$

$$
[¡!_b\varphi]B_a\psi \quad \leftrightarrow \quad \neg(B_b\varphi \vee B_b\neg\varphi) \rightarrow B_a[!_b\varphi]\psi
$$

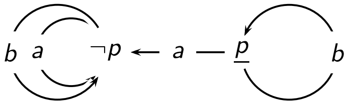
$\ldots$

When agent b is caught out as a liar

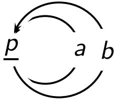
This lie is believed:



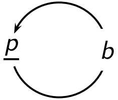
$\Rightarrow \mathrm{i}_b \neg p$



This lie is not believed:



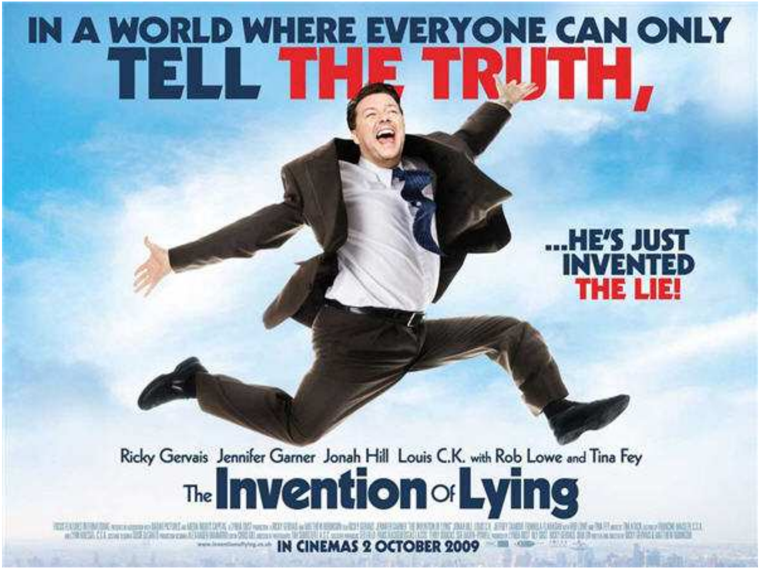
$\Rightarrow \mathrm{i}_b \neg p$



Agent b now believes ‘everything’. (There are no arrows for a.)

- Hans van Ditmarsch, The Ditmarsch Tale of Wonders — the dynamics of lying, manuscript, 2012

The invention of lying



http://www.youtube.com/watch?feature=player_detailpage&v=y

Consecutive numbers with lying

$$(0,1) - b - (2,1) - a - \underline{(2,3)} - b - (4,3) \text{———} \cdots$$

Consecutive numbers with lying

$$\begin{tikzcd}
(0,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (2,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "a"] & \underline{(2,3)} \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (4,3) \arrow[loop above, "ab"] \arrow[r, leftrightarrow] & \cdots
\end{tikzcd}$$

Consecutive numbers with lying

$$\begin{tikzcd}
(0,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (2,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "a"] & \underline{(2,3)} \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (4,3) \arrow[loop above, "ab"] \arrow[r, leftrightarrow] & \cdots
\end{tikzcd}$$

- Anne: "I know your number." Anne is lying

Consecutive numbers with lying

$$\begin{tikzcd}
(0,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (2,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "a"] & \underline{(2,3)} \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (4,3) \arrow[loop above, "ab"] \arrow[r, leftrightarrow] & \cdots
\end{tikzcd}$$

- Anne: "I know your number." Anne is lying

$$\begin{tikzcd}
(0,1) \arrow[loop above, "ab"] & (2,1) \arrow[loop above, "a"] \arrow[l, "b"'] \arrow[r, leftrightarrow, "a"] & \underline{(2,3)} \arrow[loop above, "a"] & (4,3) \arrow[loop above, "a"] \arrow[r, leftrightarrow] & \cdots
\end{tikzcd}$$

Consecutive numbers with lying

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\end{tikzcd}$$

- Bill: “That’s a lie.”

Consecutive numbers with lying

$$
\begin{tikzcd}
(0,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b" description] & (2,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "a" description] & \underline{(2,3)} \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b" description] & (4,3) \arrow[loop above, "ab"] \arrow[r, leftrightarrow] & \cdots
\end{tikzcd}
$$

Consecutive numbers with lying

$$\begin{tikzcd}
(0,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (2,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "a"] & \underline{(2,3)} \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (4,3) \arrow[loop above, "ab"] \arrow[r, leftrightarrow] & \cdots
\end{tikzcd}$$

- Anne: "I do not know your number."

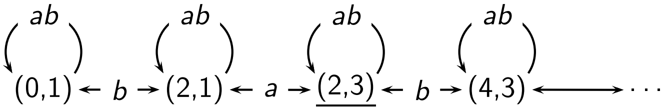
Consecutive numbers with lying

$$
\begin{tikzcd}
(0,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (2,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "a"] & \underline{(2,3)} \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (4,3) \arrow[loop above, "ab"] \arrow[r, leftrightarrow] & \cdots
\end{tikzcd}
$$

- Anne: “I do not know your number.”

$$
\begin{tikzcd}
(0,1) \arrow[loop above, "a"] \arrow[r, "b"] & (2,1) \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "a"] & \underline{(2,3)} \arrow[loop above, "ab"] \arrow[r, leftrightarrow, "b"] & (4,3) \arrow[loop above, "ab"] \arrow[r, leftrightarrow] & \cdots
\end{tikzcd}
$$

Consecutive numbers with lying



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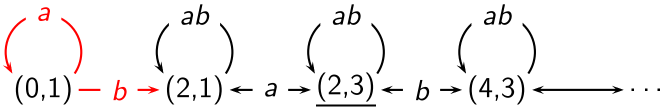

- Bill: "I know your number." Bill is lying

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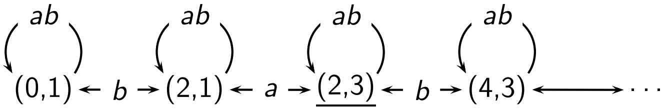
Consecutive numbers with lying



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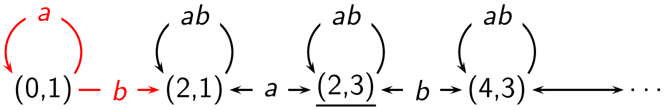

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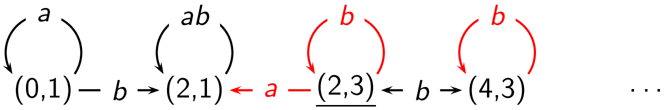

- Anne: "I know your number." Anne is mistaken.

Consecutive numbers with lying

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\end{tikzcd}$$

Action models: agent perspectives on actions

Truthful public announcement of φ:

$$\begin{tikzcd} \underline{\varphi} \arrow[loop right, "a"] \end{tikzcd}$$

Same as:

$$\begin{tikzcd} \neg\varphi \arrow[r, "a"] & \underline{\varphi} \arrow[loop right, "a"] \end{tikzcd}$$

A lie to a that φ:

$$\begin{tikzcd} \underline{\neg\varphi} \arrow[r, "a"] & \varphi \arrow[loop right, "a"] \end{tikzcd}$$

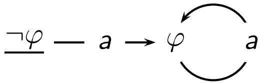
Action model for b truthtelling, lying and bluffing to a:

$$\begin{tikzcd} \neg(B_a\varphi \vee B_a\neg\varphi) \arrow[loop above, "a"] \arrow[r, "b"] & B_a\varphi \arrow[loop above, "ab"] & B_a\neg\varphi \arrow[loop above, "a"] \arrow[l, "b"'] \end{tikzcd}$$

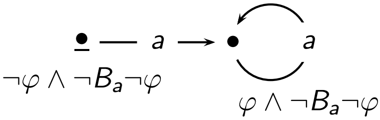
Skeptical agents and plausible information

- A *skeptical* agent does not accept new information φ if it already believes $\neg\varphi$: more complex preconditions.
- Agents may distinguish between more and less plausible states, and more and less plausible actions: a truthful announcement is more plausible than a lying announcement.

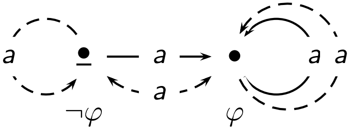
A lie to a that φ:



A lie to a skeptical agent:



The truth that φ, is more plausible than the lie that φ:



Oranges in Sevilla

p = Oranges freeze in Sevilla
a = Hans
b = you

- Truthful announcement that p: $B_a p$ and $!_a p$
- Lying announcement that p: $B_a \neg p$ and ¡$_a p$
- Bluffing announcement that p: $\neg(B_a p \vee B_a \neg p)$ and ¡!$_a p$
- Honest mistake that p: $\neg p \wedge B_a p$ en $!_a p$
- The postcondition that holds: $B_b p$
- If you are skeptical, precondition: $\neg B_b \neg p$

The Lying Game

Question-answer games with lies.
Strongly negative payoffs for being caught as a liar.

Further issues with lying

- Incorporating common knowledge/belief:

$$B_a\neg\varphi \wedge \neg B_b\neg\varphi \wedge C_{ab}((B_a\varphi \vee B_a\neg\varphi) \wedge \neg(B_b\varphi \vee B_b\neg\varphi))$$

- Insincere or strategic voting in social choice is a form of lying.
- Protocols with few liars or few lies.
- Signal analysis: noise versus intentional noise.
- Liar's Paradox in dynamic epistemic logic!

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Thank you!