Can doxastic agents learn?

On the epistemic and temporal structure of learning

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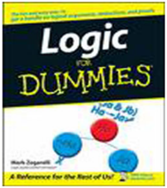
to know

to know

to know


We're not
gossiping!
We're networking!

to know


Logic
DUMMIES
A Reference for the Rest of Us!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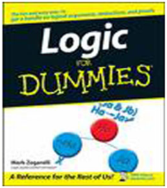
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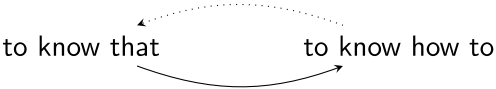
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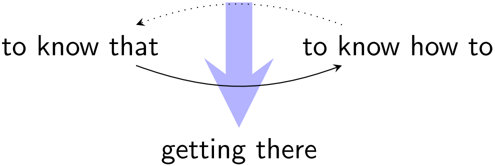
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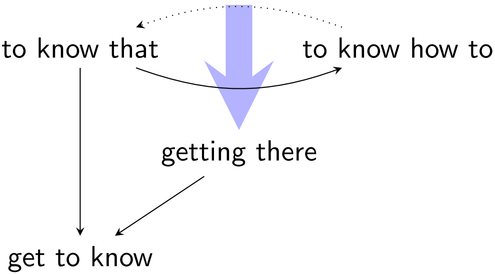
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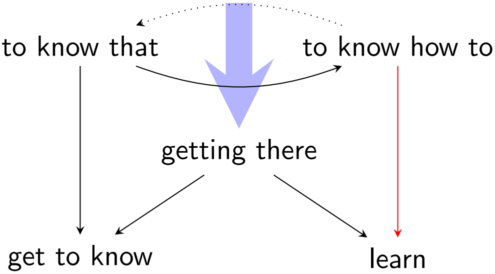
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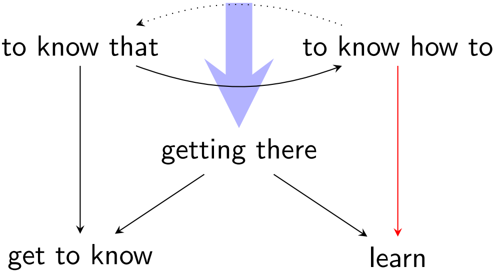
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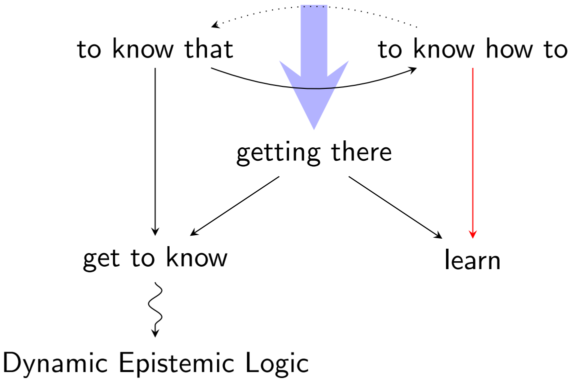
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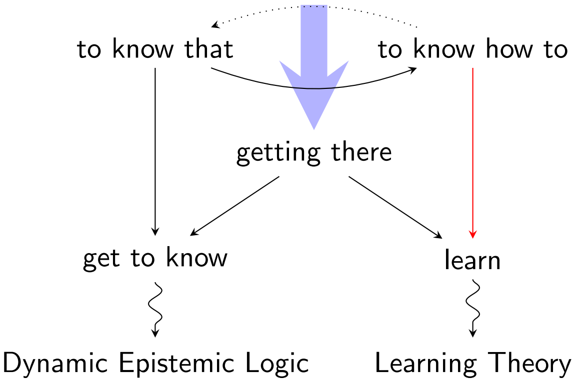
Dynamic Epistemic Logic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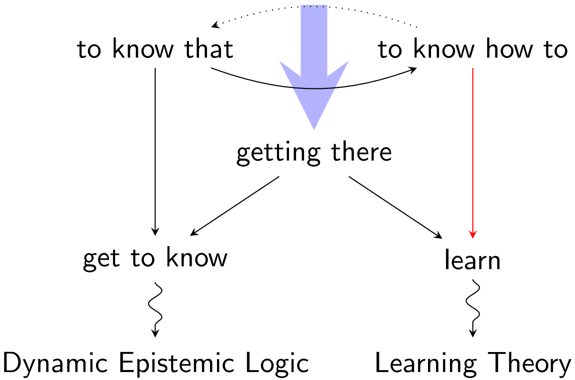
Dynamic Epistemic Logic
Learning Theory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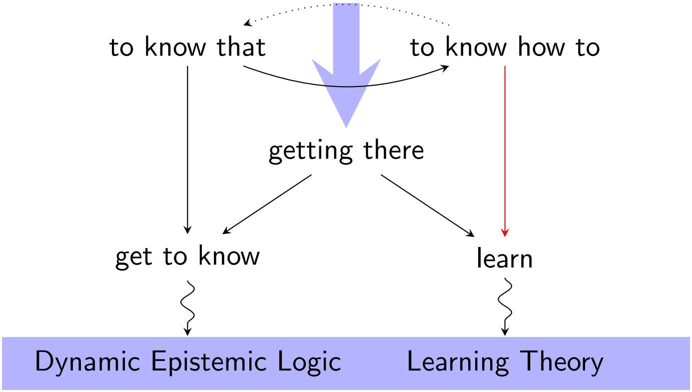
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Dynamic Epistemic Logic
Learning Theory

Outline

Outline

The Two

Formal attempts to grasp the phenomenon of epistemic change:

- formal learning theory (FLT) with scientific discovery,
- belief-revision theory and dynamic epistemic logic (DEL).

Learning Theory

Identification

1. A class of possible worlds.
2. One is the actual one (Learner does not know which).
3. Data about the world are generated.
4. From this inductively given data Learner draws his conjectures.
5. Each time: new info $\rightarrow$ Learner can answer.
6. Learner gets to a correct hypothesis.

Success Condition as a Parameter

1. Identification in the limit.
2. Finite identification.
3. Learning by erasing.

Identification in the limit

Definition

We say that a learning function $L : \mathbb{N}^* \to \mathbb{N}$:

1. identifies $S_i \in C$ in the limit on ε iff for co-finitely many m, $L(\varepsilon|m) = i$;
2. identifies $S_i \in C$ in the limit iff identifies S_i in the limit on every ε for S_i;
3. identifies C in the limit iff identifies in the limit every $S_i \in C$.

Finite identification

Definition

We say that a learning function L:

1. finitely identifies $S_i \in C$ on ε iff, when successively fed ε, at some point L outputs i, and stops;
2. finitely identifies $S_i \in C$ iff it finitely identifies S_i on every ε for S_i;
3. finitely identifies C iff it finitely identifies every $S_i \in C$.

Learning by Erasing

Definition (Function Stabilization)

Learning function stabilizes to i on environment ε iff for co-finitely many $n \in \mathbb{N}$:

$$i = min\{\mathbb{N} - \{L(\varepsilon|0), \ldots, L(\varepsilon|n)\}\}.$$

Definition

We say that a learning function L:

1. learns $S_i \in C$ by erasing on ε iff L stabilizes to i on ε;
2. learns $S_i \in C$ by erasing iff it learns by erasing S from every ε for S_i;
3. learns C by erasing iff it learns by erasing every $S_i \in C$.

Outline

The Bridge

- Initial class of languages = possible worlds;
- Relations mirror Learner's initial uncertainty and preferences;
- A world is assigned a protocol that indicates admissible sequences of events (possible environments of a language);
- Incoming piece is an event that modifies the initial model;
- Update generates a doxastic epistemic temporal forest.

The Bridge — formally

Definition (Initial epistemic model)

$\mathcal{M}_\Omega$ is a triple:

$$\langle W_\Omega, \sim_\Omega, V_\Omega \rangle,$$

where $W_\Omega = \Omega$, $\sim_\Omega = W_\Omega \times W_\Omega$, and for each set $S_i \in \Omega$, we take a nominal i and we set $V(i) = \{S_i\}$.

Definition (Single event model)

For each piece of data, we have an event model $\mathcal{E} = \langle \{e\}, \sim^{\mathcal{E}}, \mathtt{pre}_{\mathcal{E}} \rangle$ where $\sim^{\mathcal{E}} = \{(e, e)\}$ and $\mathtt{pre}_{\mathcal{E}}(e) = \top$.

Definition (Local protocol of $(\mathcal{M}_\Omega, S_i)$)

Given a state $S_i \in W_\Omega$, our protocol P_Ω should authorize at S_i any ω-sequence that enumerates S_i and nothing more.

FOUR WAYS

- Semantic properties of learning as iterated update.
- Modal characterizations of forests generated by learning.
- Learnability conditions as properties of temporal models.
- DETL counterparts of FLT characterization theorems.

Outline

DEL and learning problems

Definition (Stabilization of iterated update)

Iterated epistemic update of model $\mathcal{M}$ with an infinite sequence of events ϵ stabilizes to $\mathcal{M}'$ iff $\exists n \in N\ \forall m \geq n,\ \mathcal{M}^{\epsilon|m} = \mathcal{M}'$.

Theorem

The following are equivalent:

1. *Ω is finitely identifiable.*
2. *For all $S_i \in W_\Omega$ and $\epsilon \in P_\Omega(S_i)$ the generated epistemic model $\mathcal{M}^{\epsilon}_{\Omega}$ stabilizes to $\mathcal{M}'_\Omega = \langle W'_\Omega, \sim'_\Omega, V_\Omega \rangle$, where $W'_\Omega = \{S_i\}$ and $\sim'_\Omega = \{(S_i, S_i)\}$.*
3. *For all $S_i \in W_\Omega$ and $\epsilon \in P_\Omega(S_i)$ the generated epistemic model $\mathcal{M}^{\epsilon}_{\Omega}$ stabilizes to $\mathcal{M}'_\Omega = \langle W'_\Omega, \sim'_\Omega, V_\Omega \rangle$, where $W'_\Omega = \{S_i\}$ and $\mathcal{M}'_\Omega, S_i \Vdash K\ i$.*

DEL and ETL

Theorem (van Benthem et al. 2009)

An ETL-model $\mathcal{H}$ is isomorphic to the forest generated by the sequential product update of an epistemic model according to some state-dependent DEL-protocol iff it satisfies perfect recall, synchronicity, uniform no miracles and propositional stability.

Language of our hybrid DETL

$$\varphi := p \mid i \mid x \mid \downarrow x.\varphi \mid \neg\varphi \mid \varphi \vee \varphi \mid K_j\varphi \mid \mathtt{A}\varphi \mid \bigcirc^{-1}\varphi \mid F\varphi \mid P\varphi \mid \forall\varphi$$

$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash p$	iff	$w\vec{e} \in V(p)$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash i$	iff	$V(i) = w$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash x$	iff	$g(x) = w\vec{e}$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash \downarrow x.\phi$	iff	$\mathcal{H}, w\epsilon, w\vec{e}, g[x := w\vec{e}] \Vdash \phi$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash K_i\phi$	iff	$\forall v\vec{f}\ \forall w\epsilon$ with $v\vec{f} \in \mathcal{K}_i[w\vec{e}]$ & $v\vec{f} \sqsubseteq v\epsilon'$ we have $\mathcal{H}, v\epsilon', v\vec{f} \Vdash \phi$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash B_i\phi$	iff	$\forall v\vec{f}\ \forall w\epsilon$ with $v\vec{f} \in \mathcal{B}_i[w\vec{e}]$ & $v\vec{f} \sqsubseteq v\epsilon'$ we have $\mathcal{H}, v\epsilon', v\vec{f} \Vdash \phi$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash \mathtt{A}\phi$	iff	$\forall v\vec{f}\ \forall w\epsilon$ with $v\vec{f} \in H$ & $v\vec{f} \sqsubseteq v\epsilon'$ we have $\mathcal{H}, v\epsilon', v\vec{f} \Vdash \phi$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash \bigcirc^{-1}\phi$	iff	$\exists a \in \Sigma\ \exists \vec{f} \sqsubseteq \epsilon$ such that $\vec{f}.a = \vec{e}$ and $\mathcal{H}, w\epsilon, w\vec{f} \Vdash \phi$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash F\phi$	iff	$\exists \vec{g} \in \Sigma^*\ \exists \vec{f} \sqsubseteq \epsilon$ such that $\vec{f} = \vec{e}\vec{g}$ and $\mathcal{H}, w\epsilon, w\vec{f} \Vdash \phi$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash P\phi$	iff	$\exists \vec{g} \in \Sigma^*\ \exists \vec{f} \sqsubseteq \epsilon$ such that $\vec{f}\vec{g} = \vec{e}$ and $\mathcal{H}, w\epsilon, w\vec{f} \Vdash \phi$
$\mathcal{H}, w\epsilon, w\vec{e}, g \Vdash \forall\phi$	iff	$\forall h' \in \mathfrak{P}(w)$ s.t. $\vec{e} \sqsubseteq h$ we have $\mathcal{H}, wh', w\vec{e} \Vdash \phi$

Hybrid DETL characterizations of learning

Theorem

The following are equivalent:

1. Ω *is finitely identifiable.*
2. *For all* $s \in W_\Omega$ *and* $\epsilon \in P_\Omega(s)$ *the learner's knowledge about the initial state stabilizes to* s *on* $s\epsilon$ *in the generated forest* $For(\mathcal{M}_\Omega, V_\Omega, P_\Omega)$.
3. $For(\mathcal{M}_\Omega, V_\Omega, P_\Omega) \Vdash \mathtt{A}(\bigcirc^{-1}\bot \rightarrow \downarrow x.\forall FKH(\bigcirc^{-1}\bot \rightarrow x))$.

Outline

DETL models for learnability
FIN

Definition

An ETL frame $F(\mathcal{H}) = \langle W, \Sigma, H, \sim_L \rangle$ satisfies Finite Identification (FIN) iff for all $s \in W$ and $h = s\epsilon \in P(s)$ Learner's *knowledge* about the initial state stabilizes to s on $s\epsilon$.

An ETL frame $F(\mathcal{H})$ satisfies FIN iff $F(\mathcal{H}) \Vdash i \rightarrow \forall F K i$

DETL models for learnability
ERASE

Definition

An ETL frame $F(\mathcal{H}) = \langle W, \Sigma, H, \sim_L \rangle$ satisfies Learning by Erasing wrt $\leq_L$, ($\leq_L$-ERASE) iff for all $s \in W$ and $h = s\epsilon \in P(s)$ Learner's *belief* about the initial state stabilizes to s on $s\epsilon$.

An ETL frame F($\mathcal{H}$) satisfies $\leq$-ERASE iff

$$F(\mathcal{H}[\leq]) \Vdash i \rightarrow \forall FGBi$$

DETL models for learnability

LIM and expressibility problems

An ETL frame F($\mathcal{H}$) satisfies ERASE iff

$$\exists \leq F(\mathcal{H}[\leq]) \Vdash i \rightarrow \forall FGBi$$

An ETL frame F($\mathcal{H}$) satisfies LIM iff

$$\exists \ \mathfrak{B}\text{-Algorithm } F(\mathcal{H}[\mathfrak{B}]) \Vdash i \rightarrow \forall FGBi$$

Outline

Conclusions and perspectives

- Semantic grasp of inductive learning in DEL.
- Learnability as a validity problem of DETL.

Some further directions:

1. Extensions: identification of functions, complete information.
2. Effects of various restrictions on protocols.
3. Constraints on learning functions and on epistemic agents.
4. Operational concept of ‘stable belief’.

谢谢！