Learning and Teaching as a Game
A Sabotage Approach

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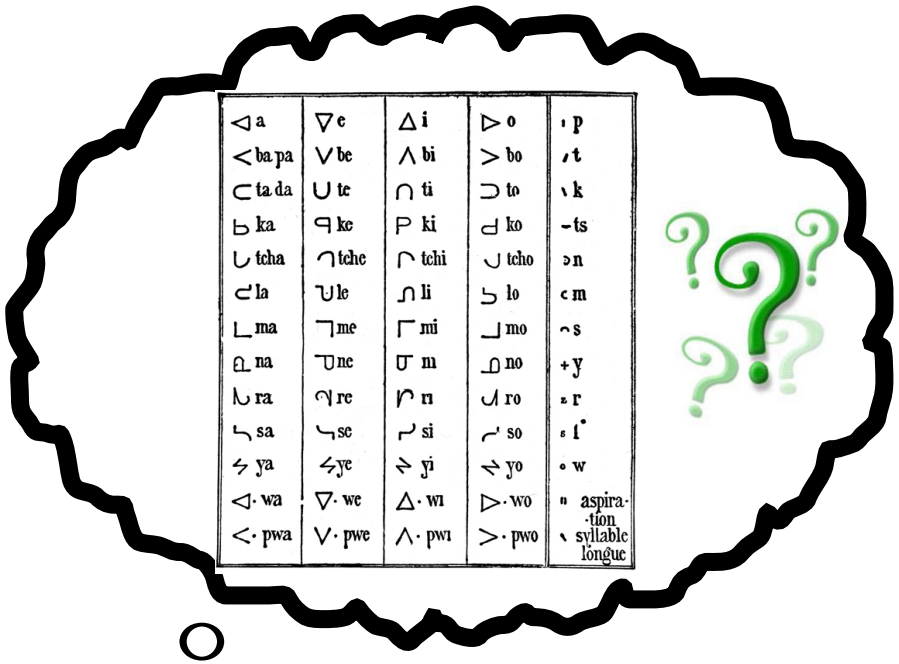
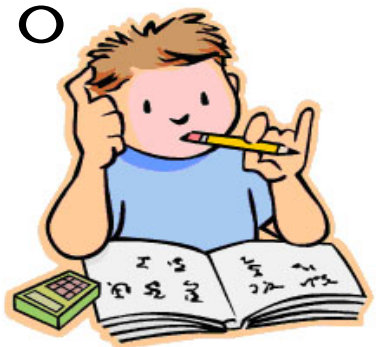
LORI'09, October 8th 2009, Chongqing

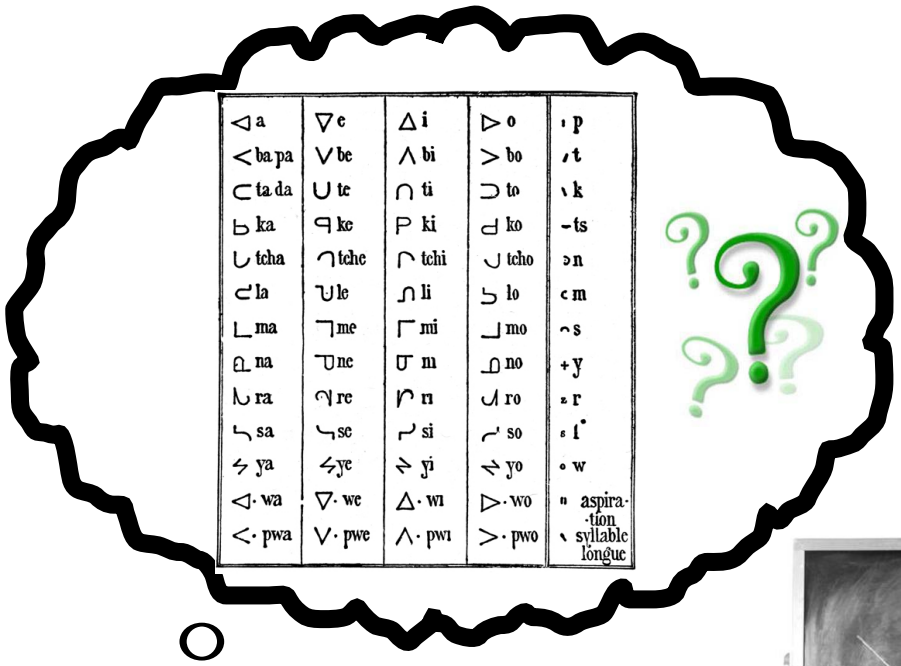
Outline

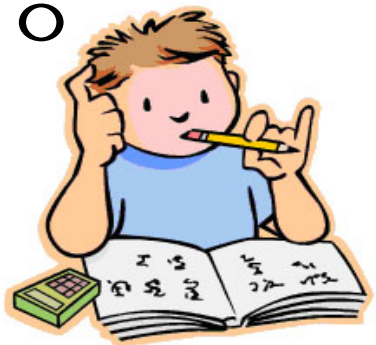
Motivation: Learning and Interaction





Motivation: Learning and Interaction



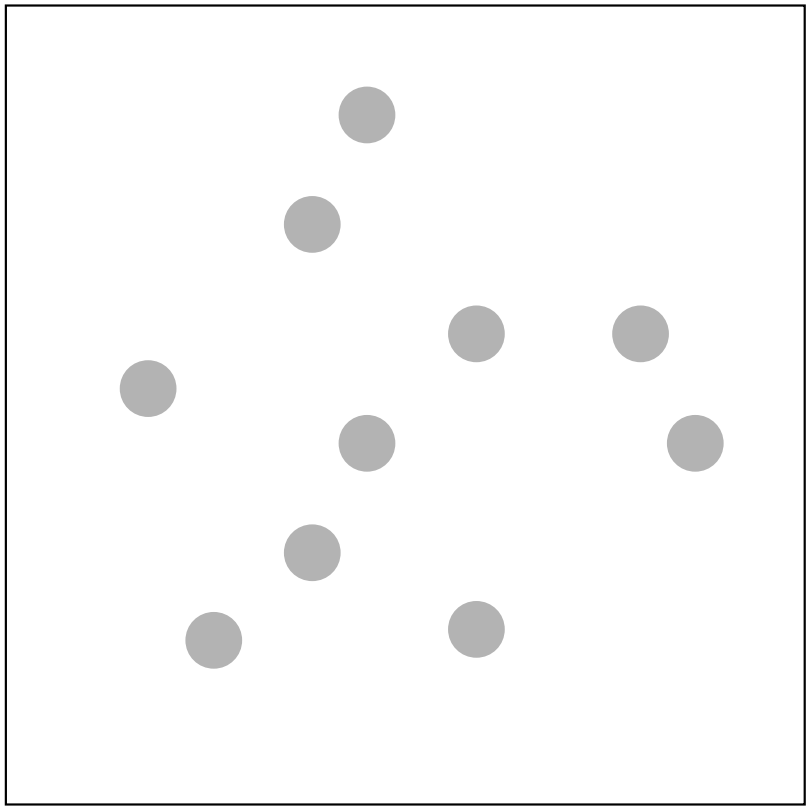


Motivation: Learning Theory

Learning as a convergence to a correct hypothesis;
on the basis of inductively given data.

- Language acquisition.
- Scientific inquiry.
- Inductive inference games.

Illustration



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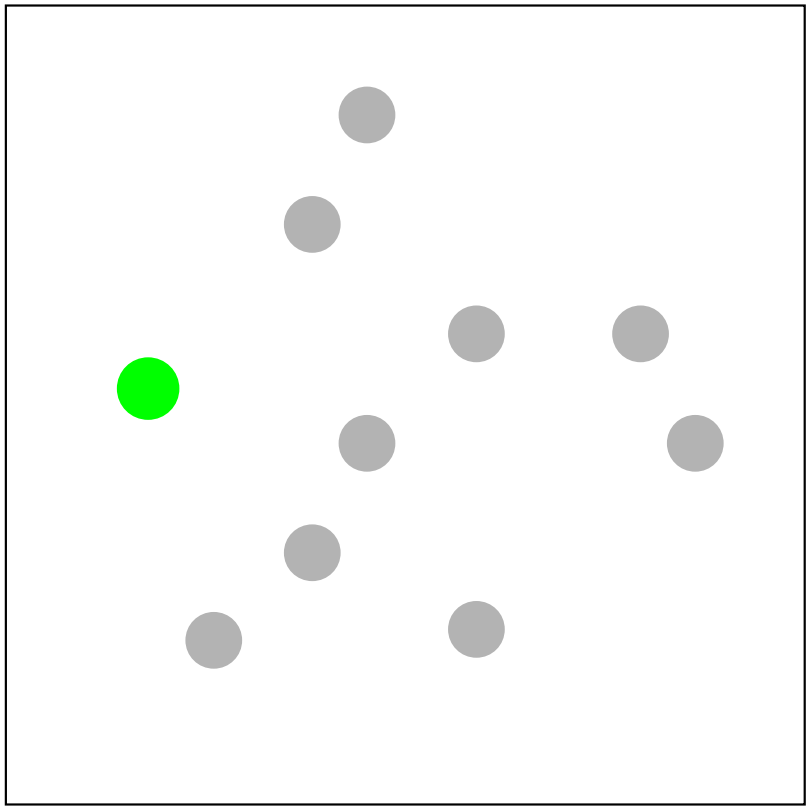

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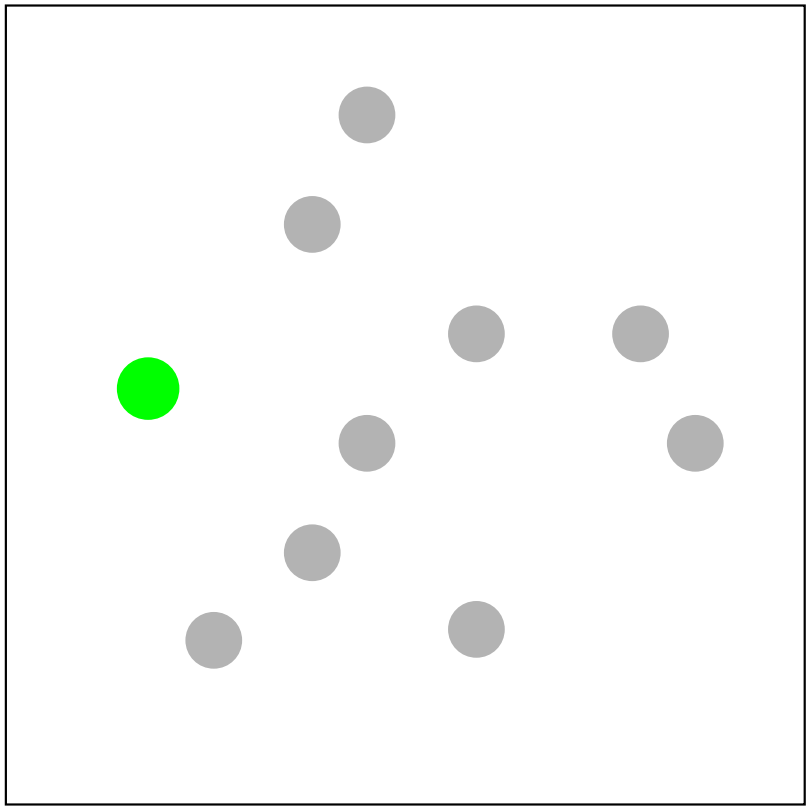
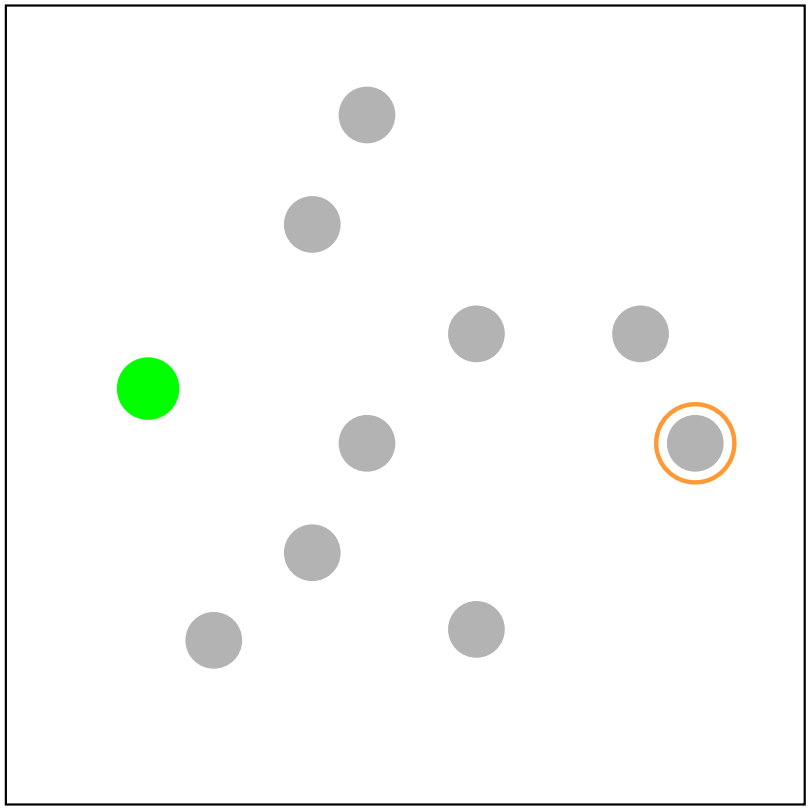


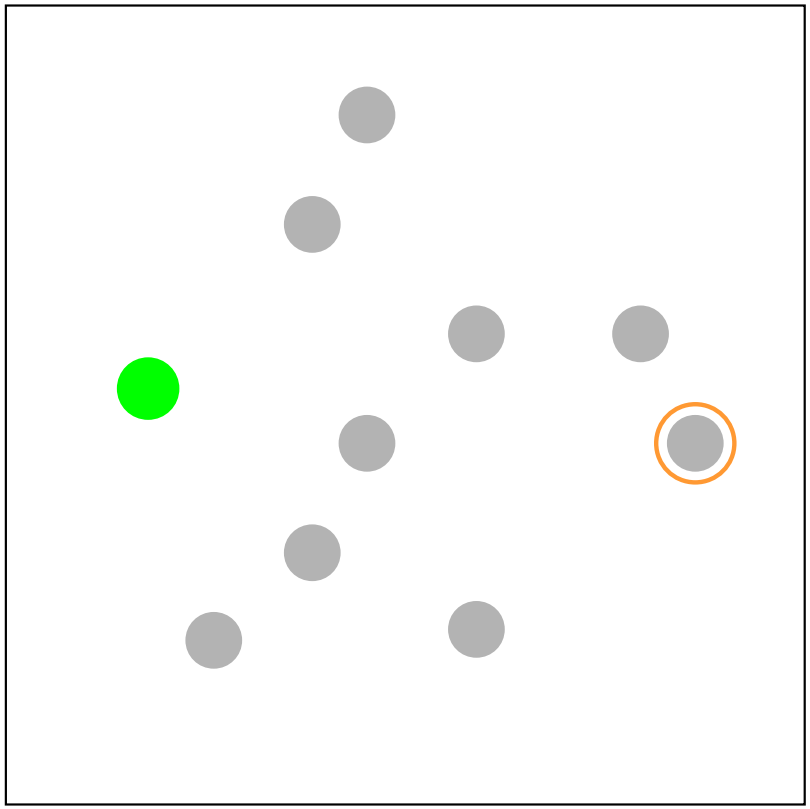
d_0

Illustration



d_0

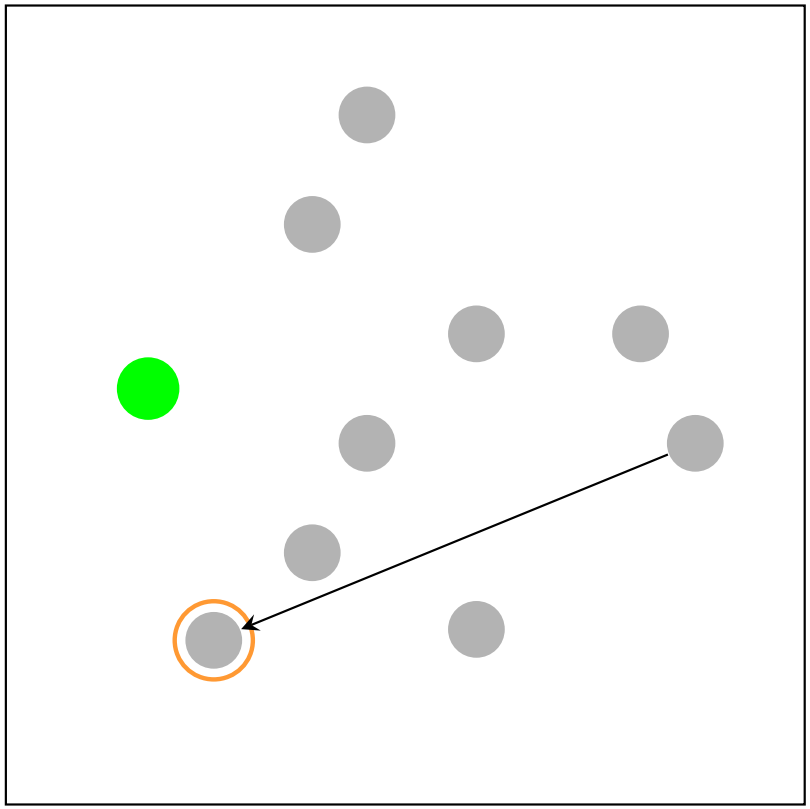
Illustration



d_1

d_0

Illustration



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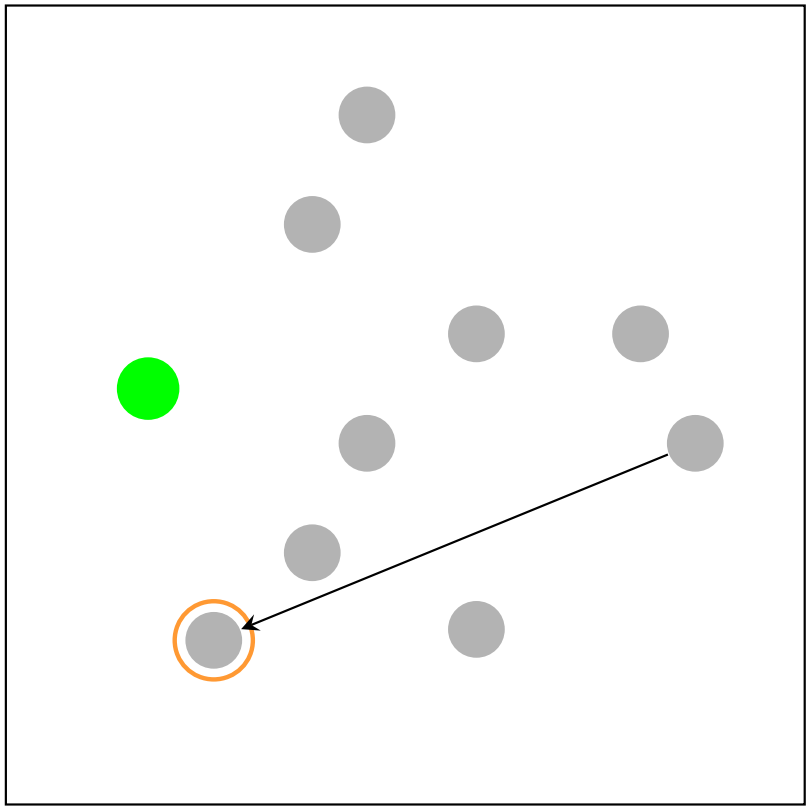

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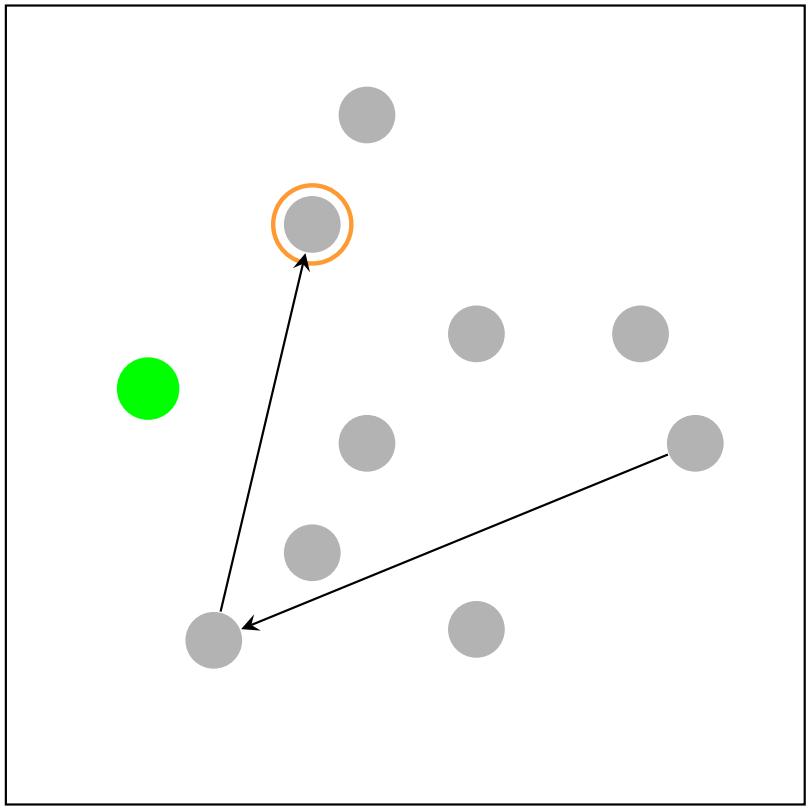


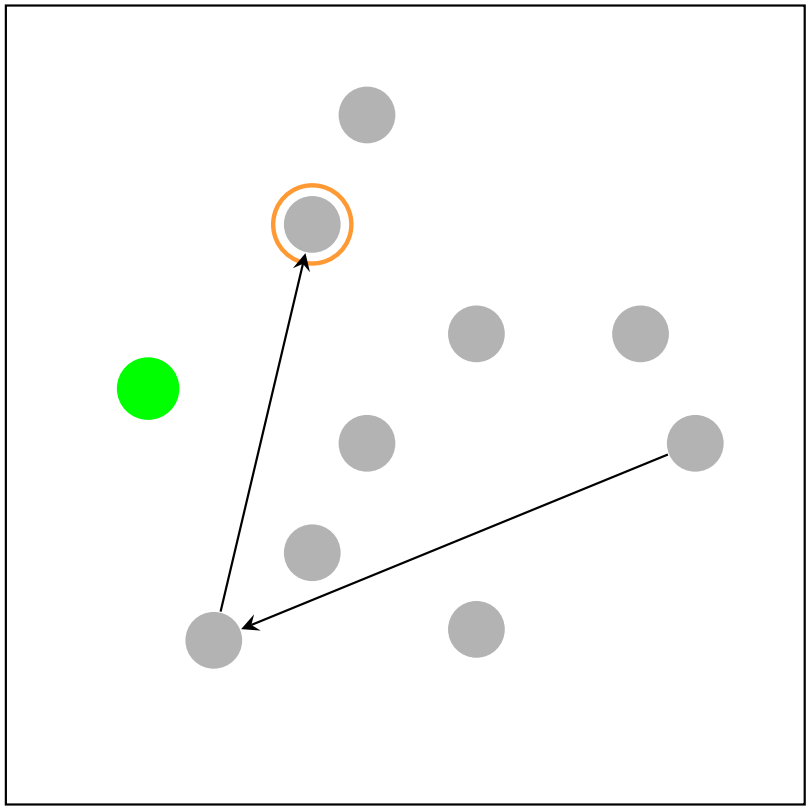
Illustration

d_3

d_2

d_1

d_0



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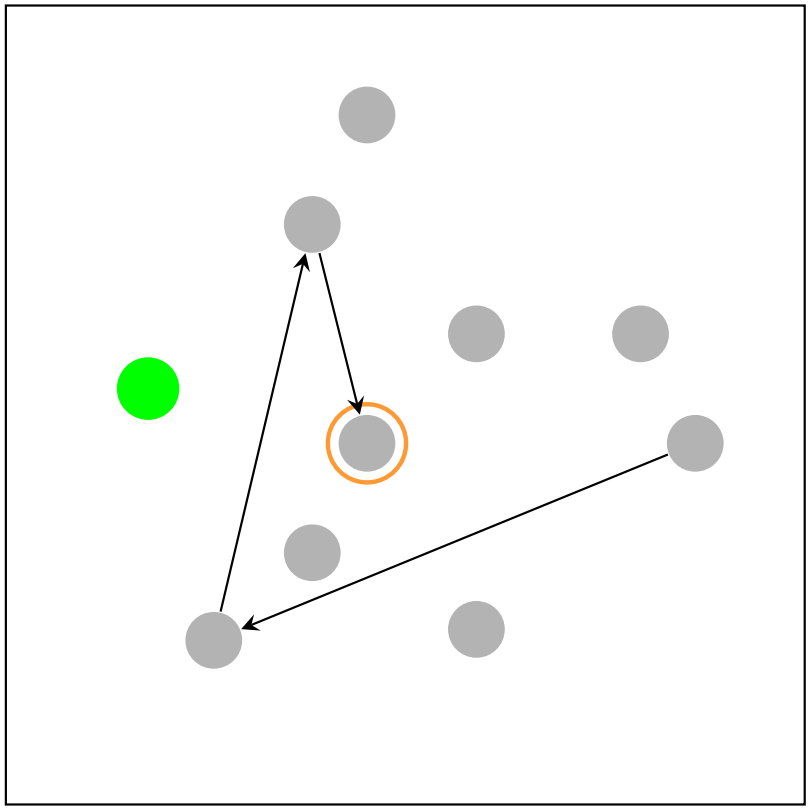
d_3

d_2

d_1

d_0



Illustration

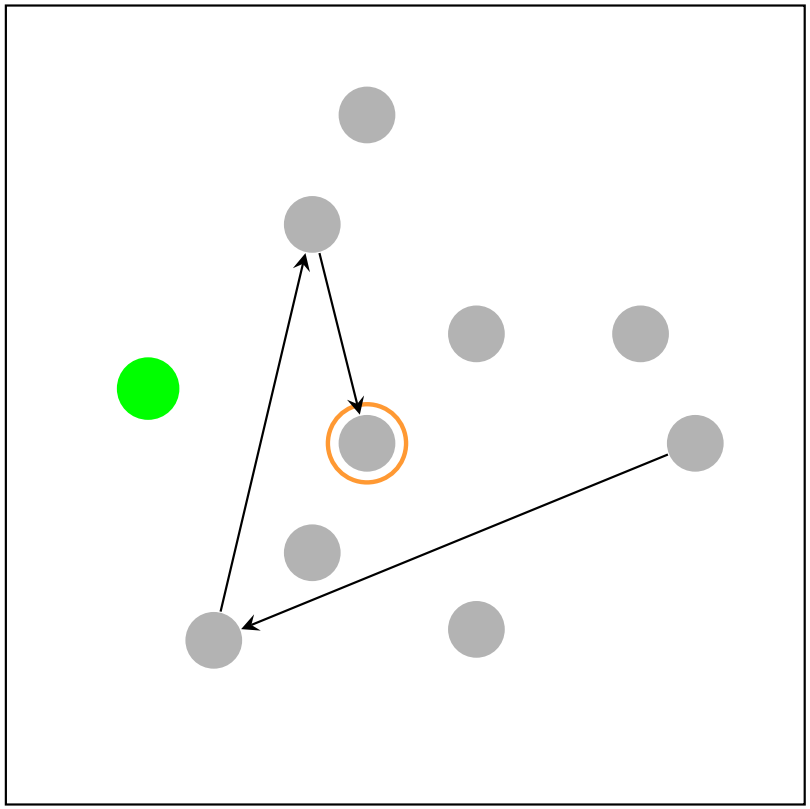
d_4

d_3

d_2

d_1

d_0



Illustration

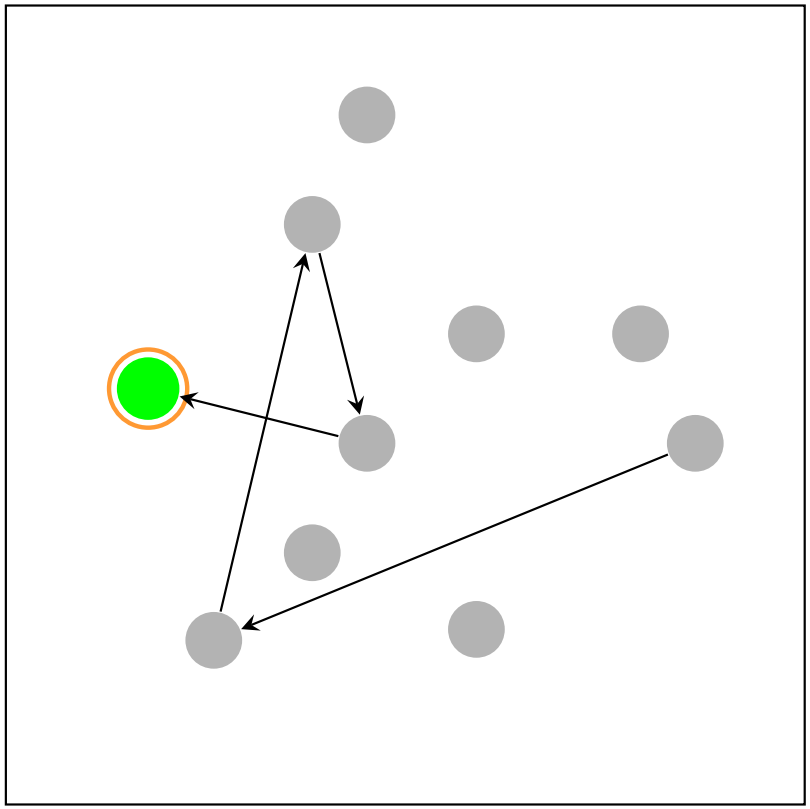
d_4

d_3

d_2

d_1

d_0



Illustration

$\ldots$

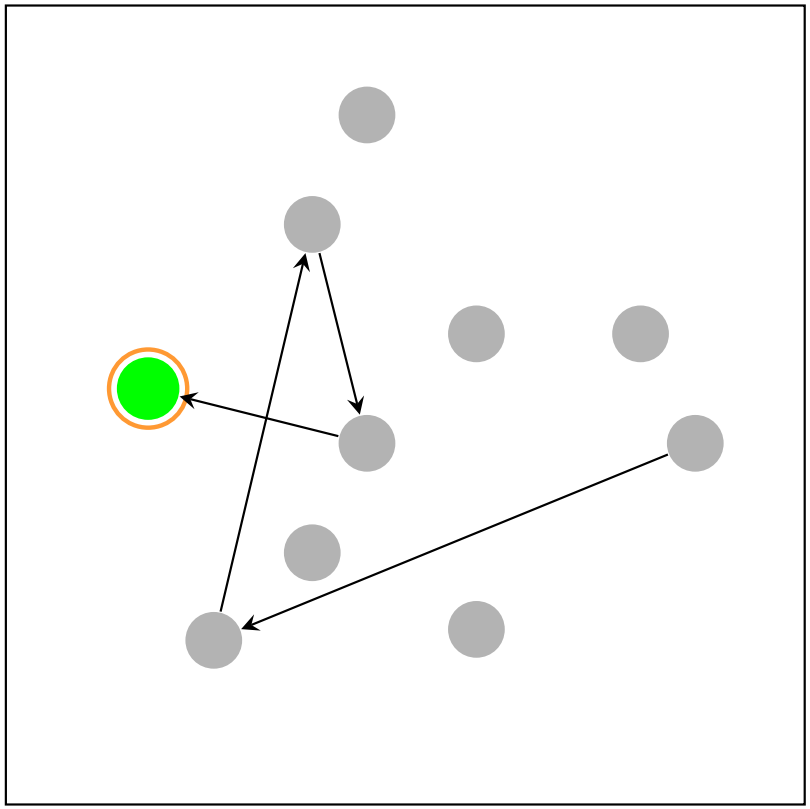
d_4

d_3

d_2

d_1

d_0



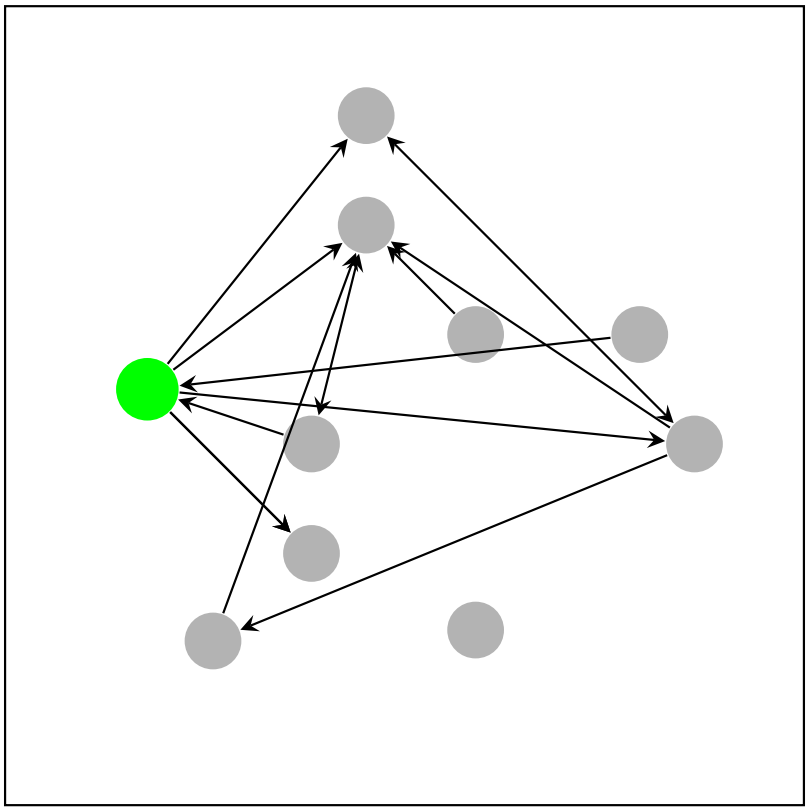
Motivation: Finite and Limiting

The main distinction:

- Finite identification — Learner is sure and stops inquiring.
- Identification in the limit — Learner keeps inquiring.
  Success iff co-finitely many answers are correct.

We focus on enforcing the finiteness of learning.

Motivation: Back to the scenario!



A class of hypotheses, with one distinguished goal and Learner generates a map of possible mind changes of Learner.

Motivation: Learning as Interaction/Game

- Learning scenarios as graphs.
- Learning as a game between Teacher(s) and Learner(s) played on a graph.
  - different moves/objectives/information;
  - players' strategies, preferences, cooperation, epistemic status.

Learning can be seen as ...

- A step-by-step process in which Learner changes his state.
- Successful if goal is eventually reached.
- Teacher's feedback rules out possible changes of mind.

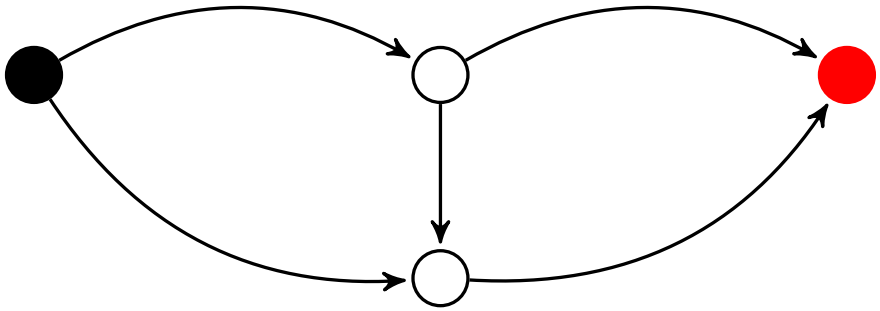
Learning as a graph-game (1)

Learning Model	**Our graph**
hypotheses	states
correct hypothesis	goal state
possibility of a mind change from hypothesis a to hypothesis b	edge from state a to b

Learning as a graph-game (2)

Learning Model	Our Games
a mind change from hypothesis a to hypothesis b	transition from state a to b
giving a counterexample that eliminates the possibility of a mind change from a to b	removing a transition between a and b

That is ...



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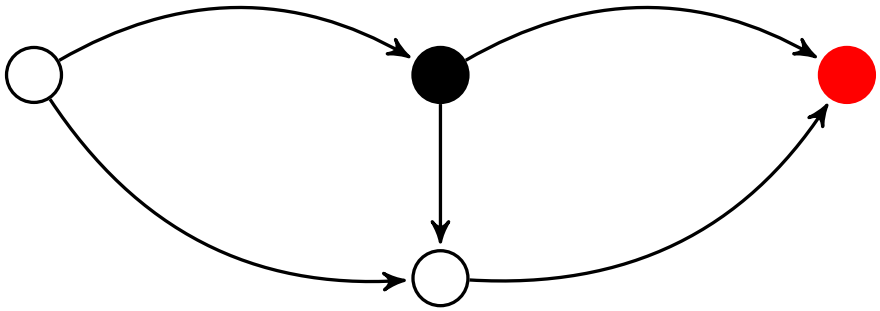

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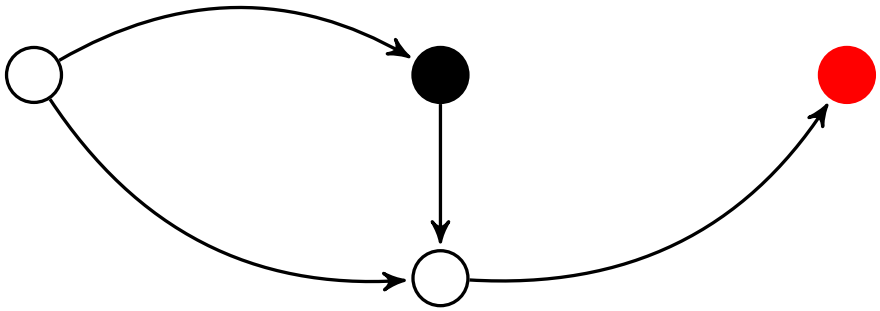

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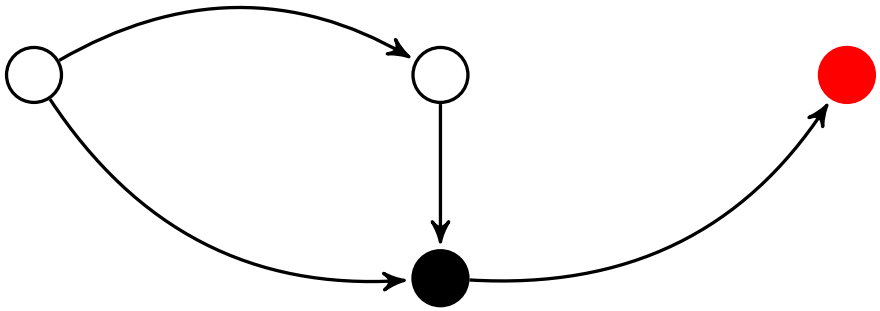

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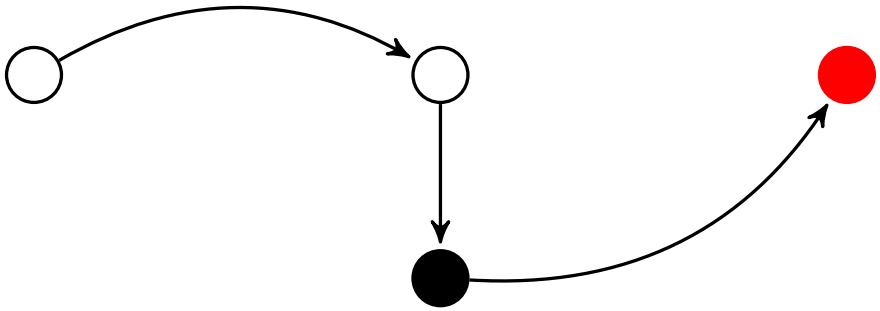

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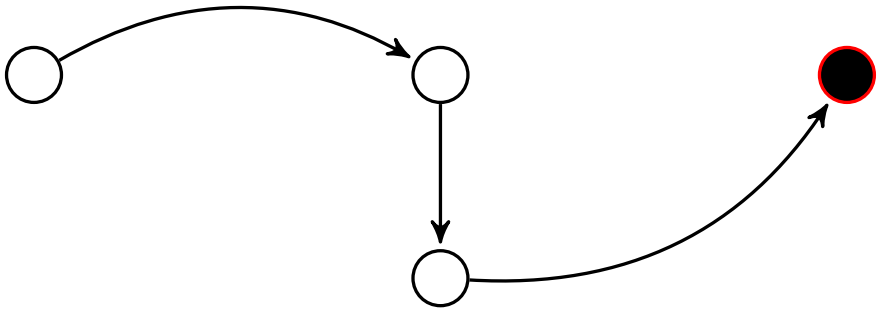
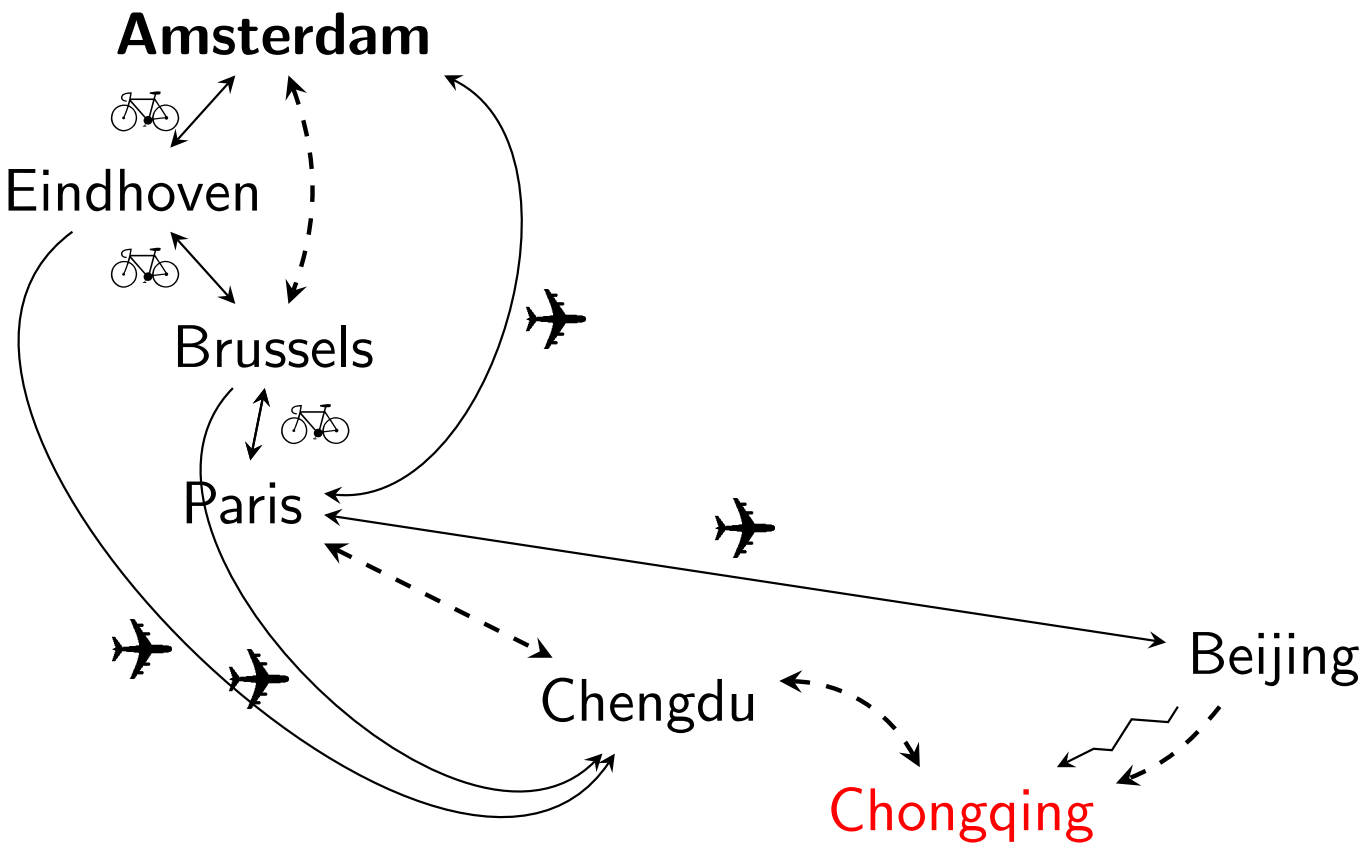


Introducing the Sabotage game

But now, the process looks similar to the one that takes place in *Sabotage Games* ...

Sabotage Game: Traveling to LORI 2009



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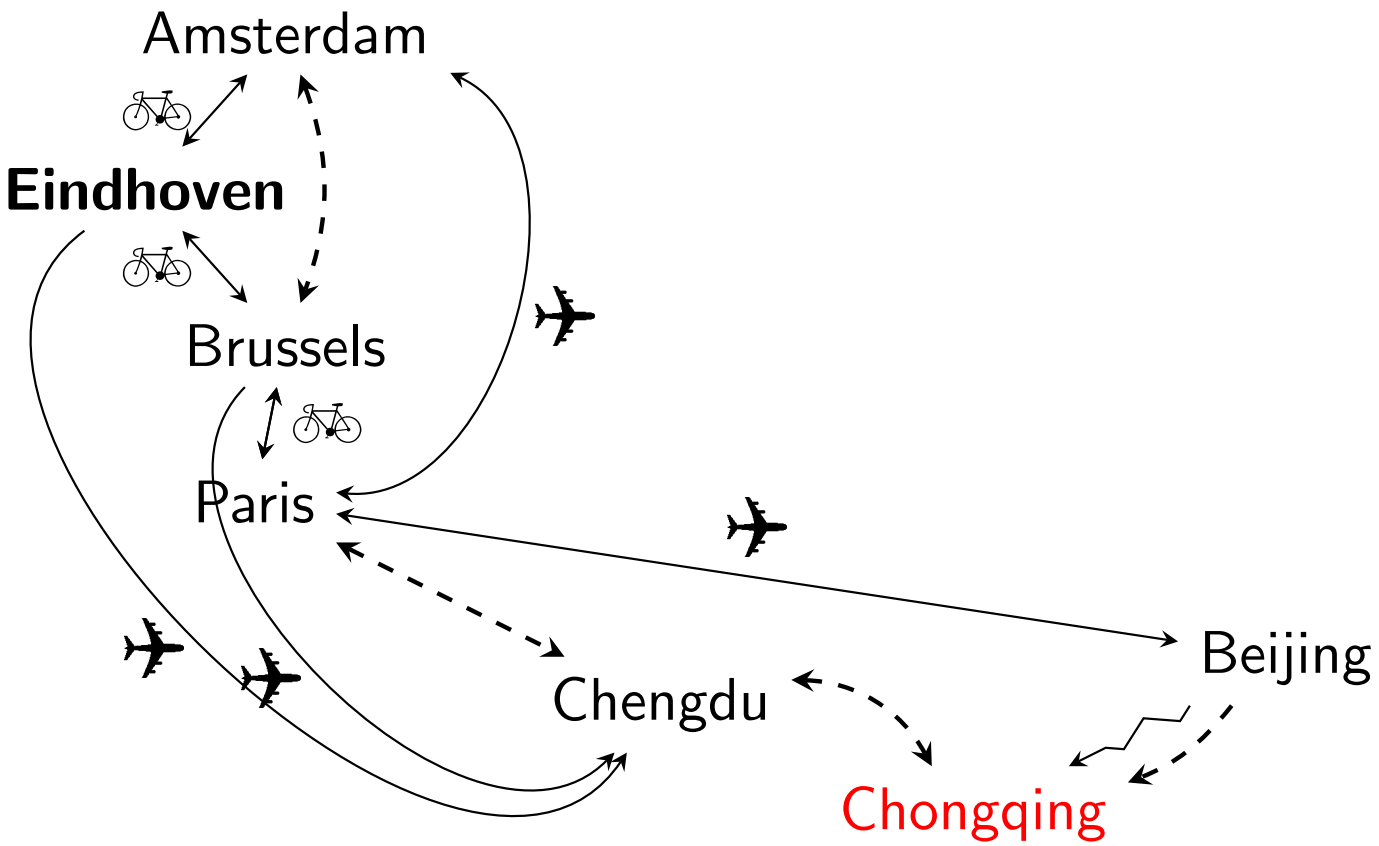

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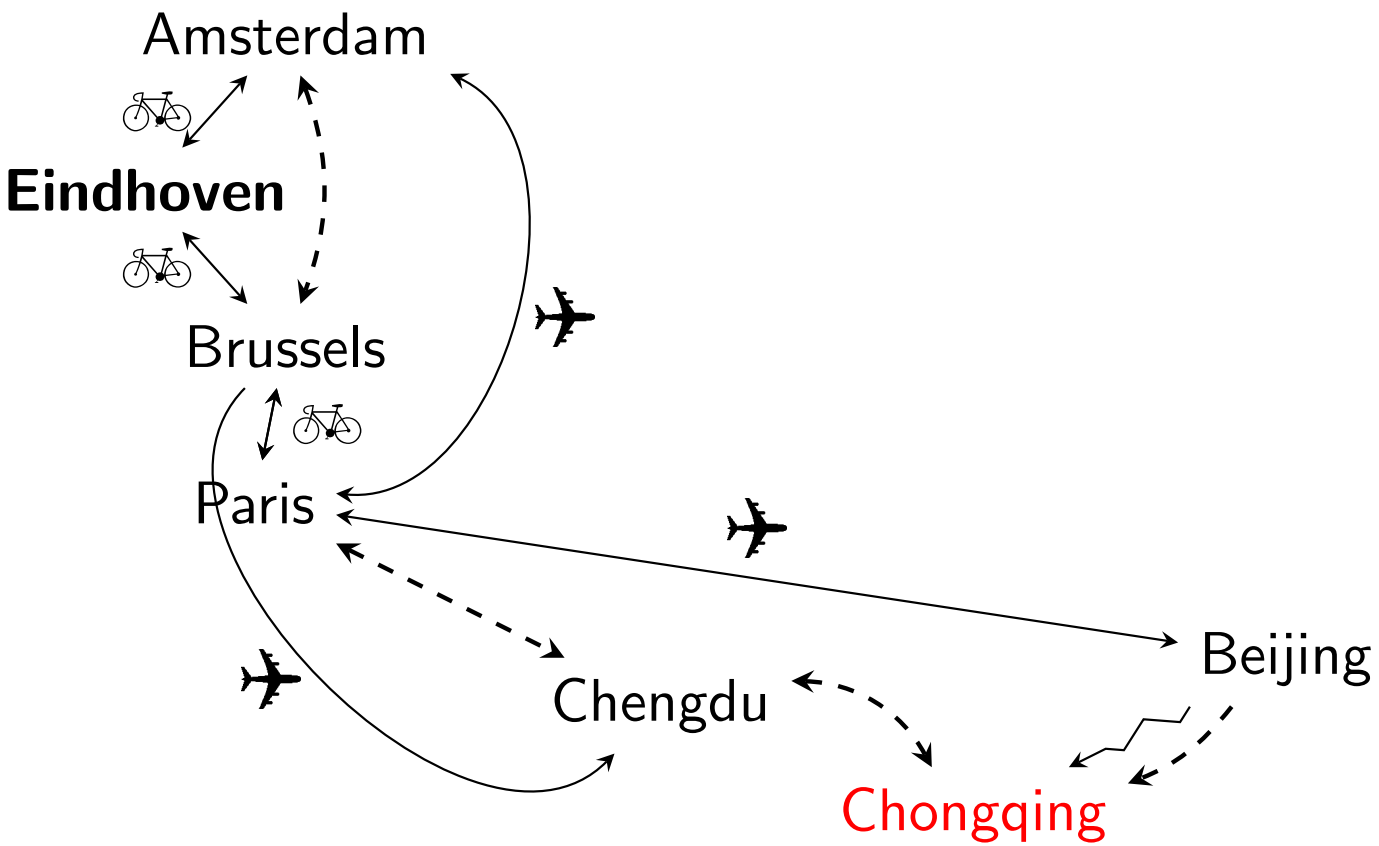

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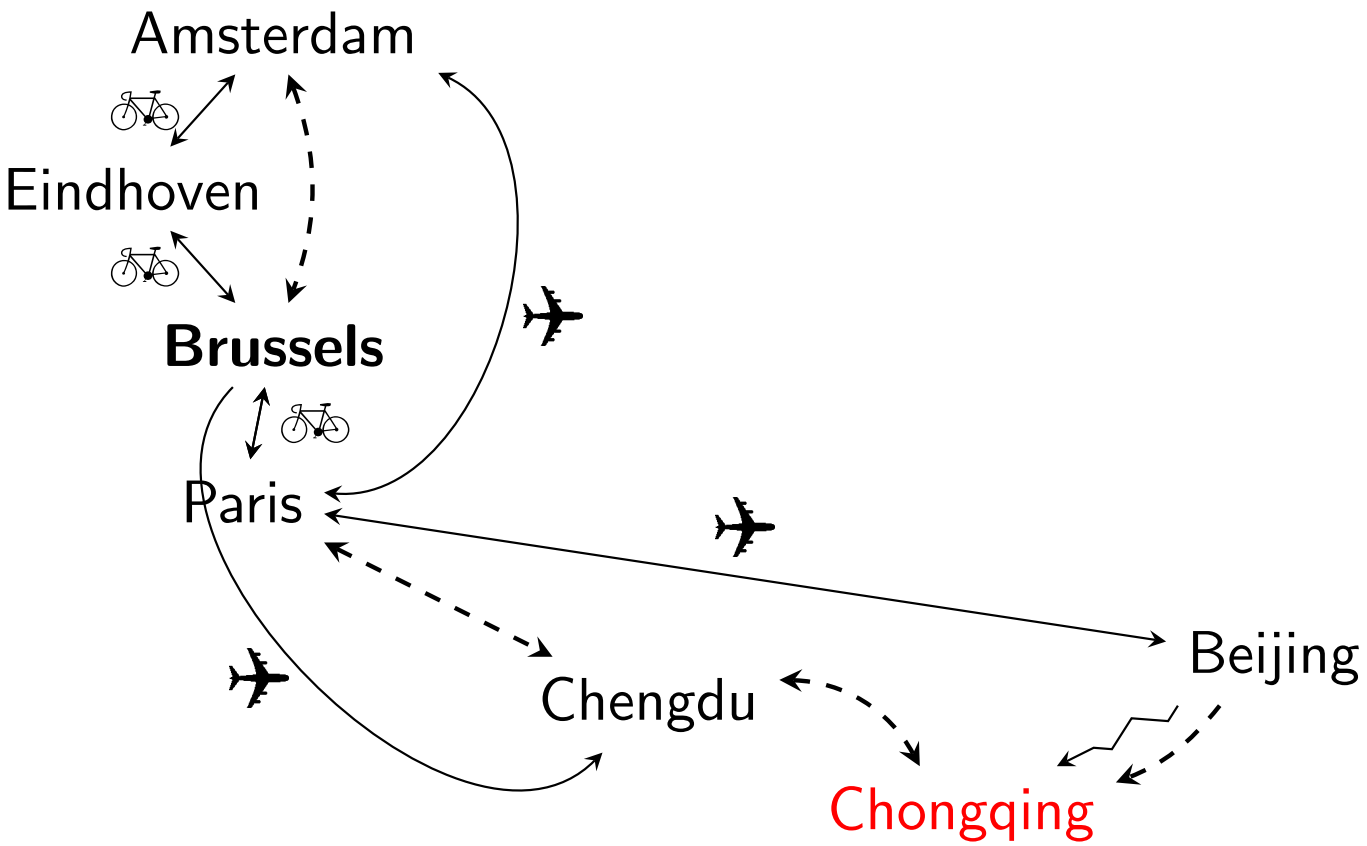

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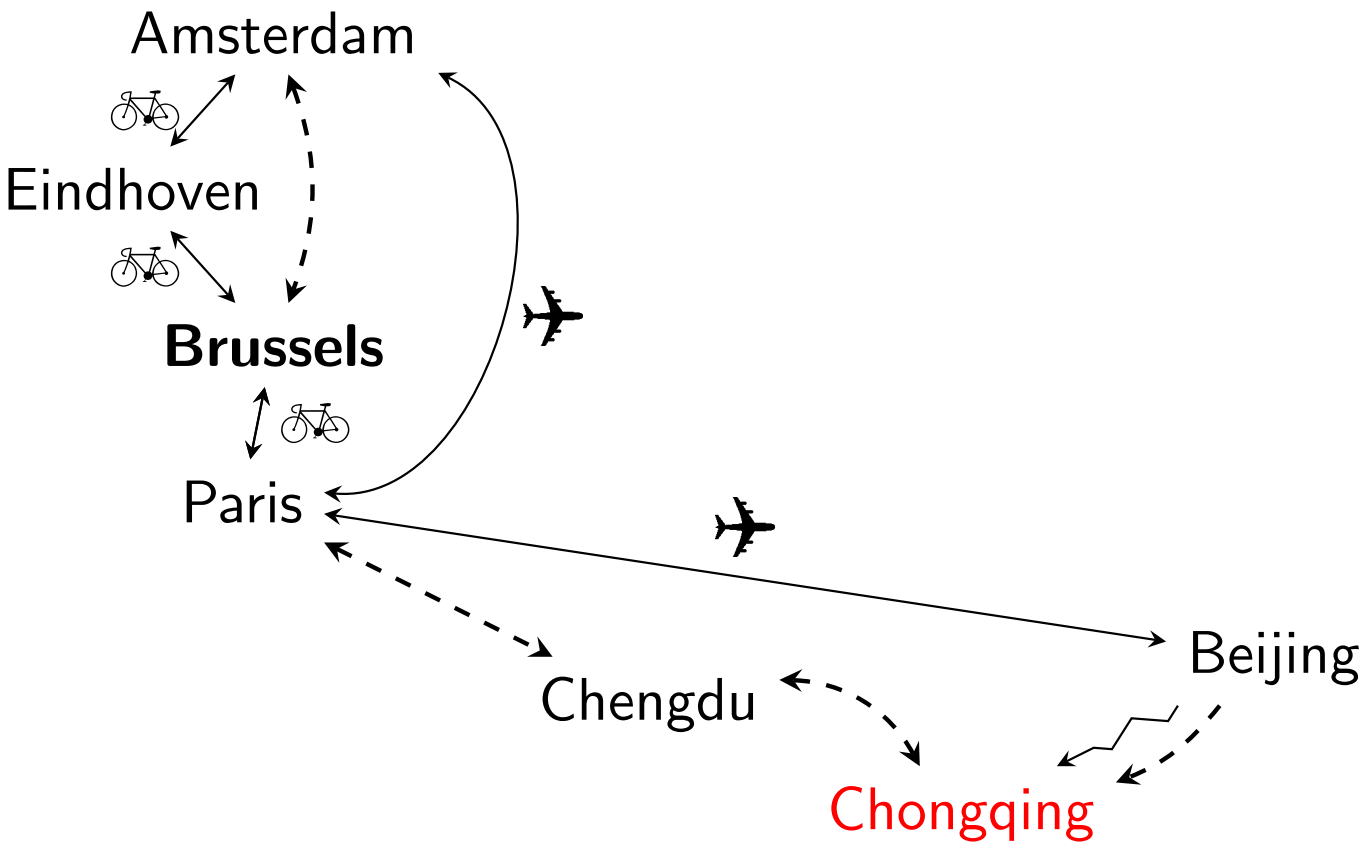

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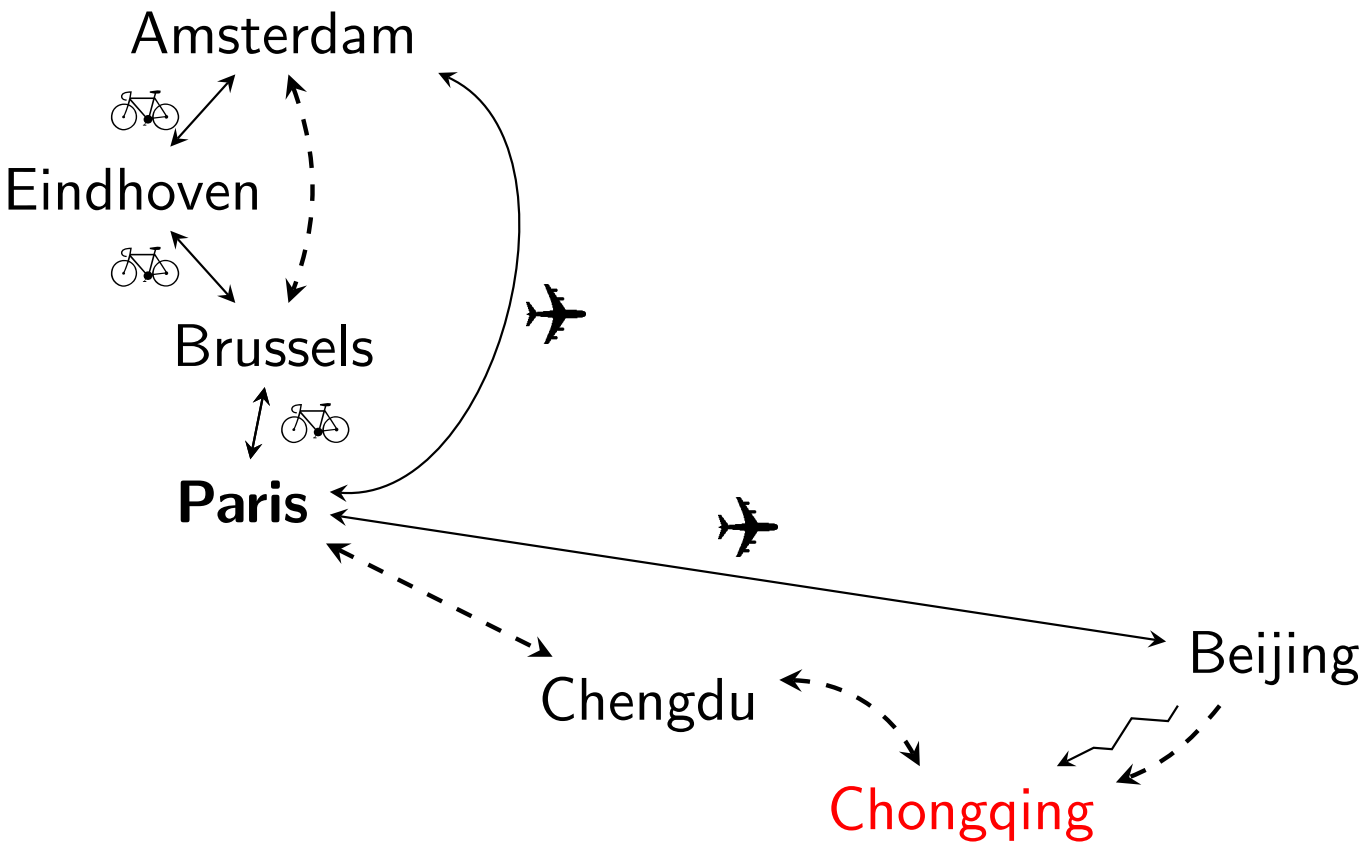

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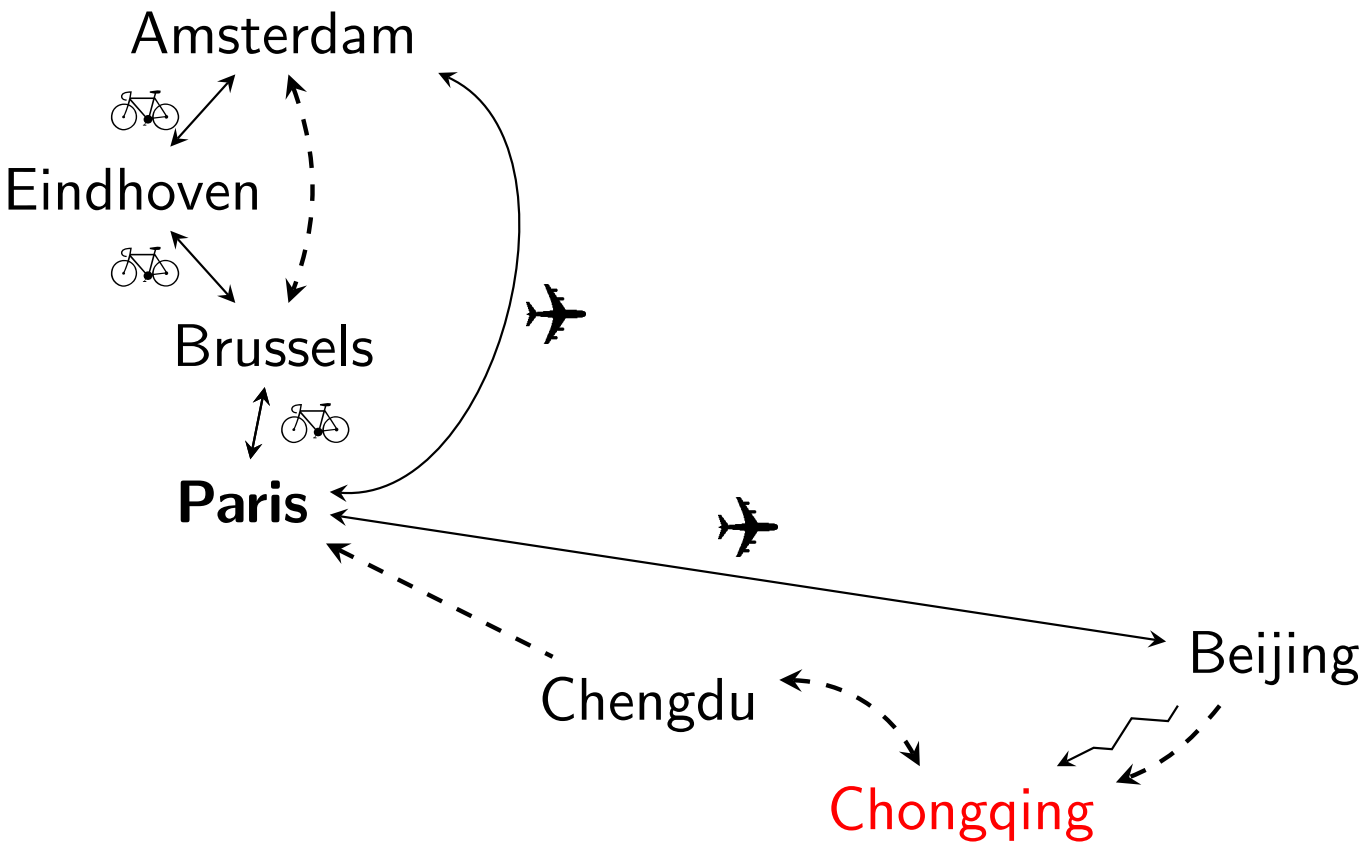

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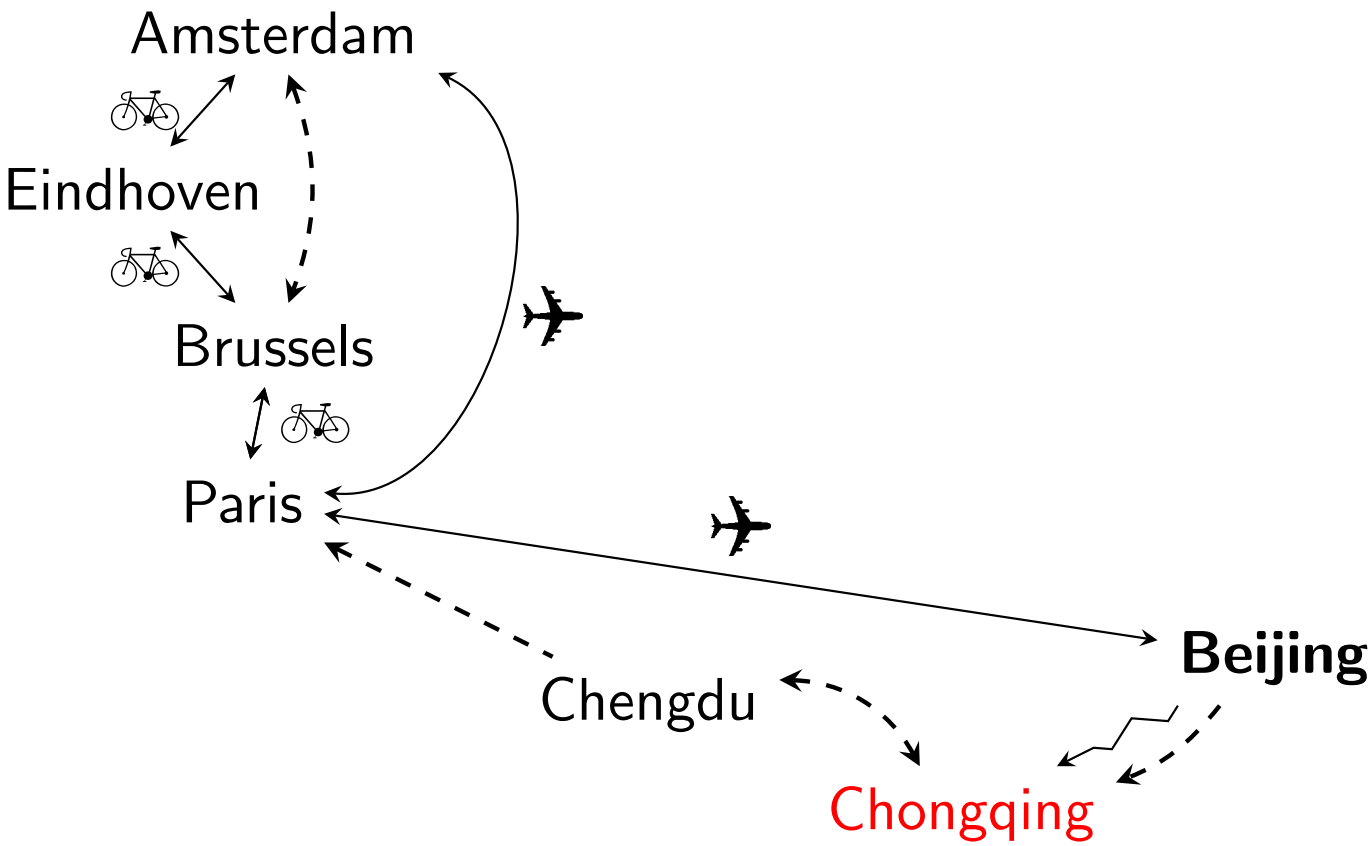

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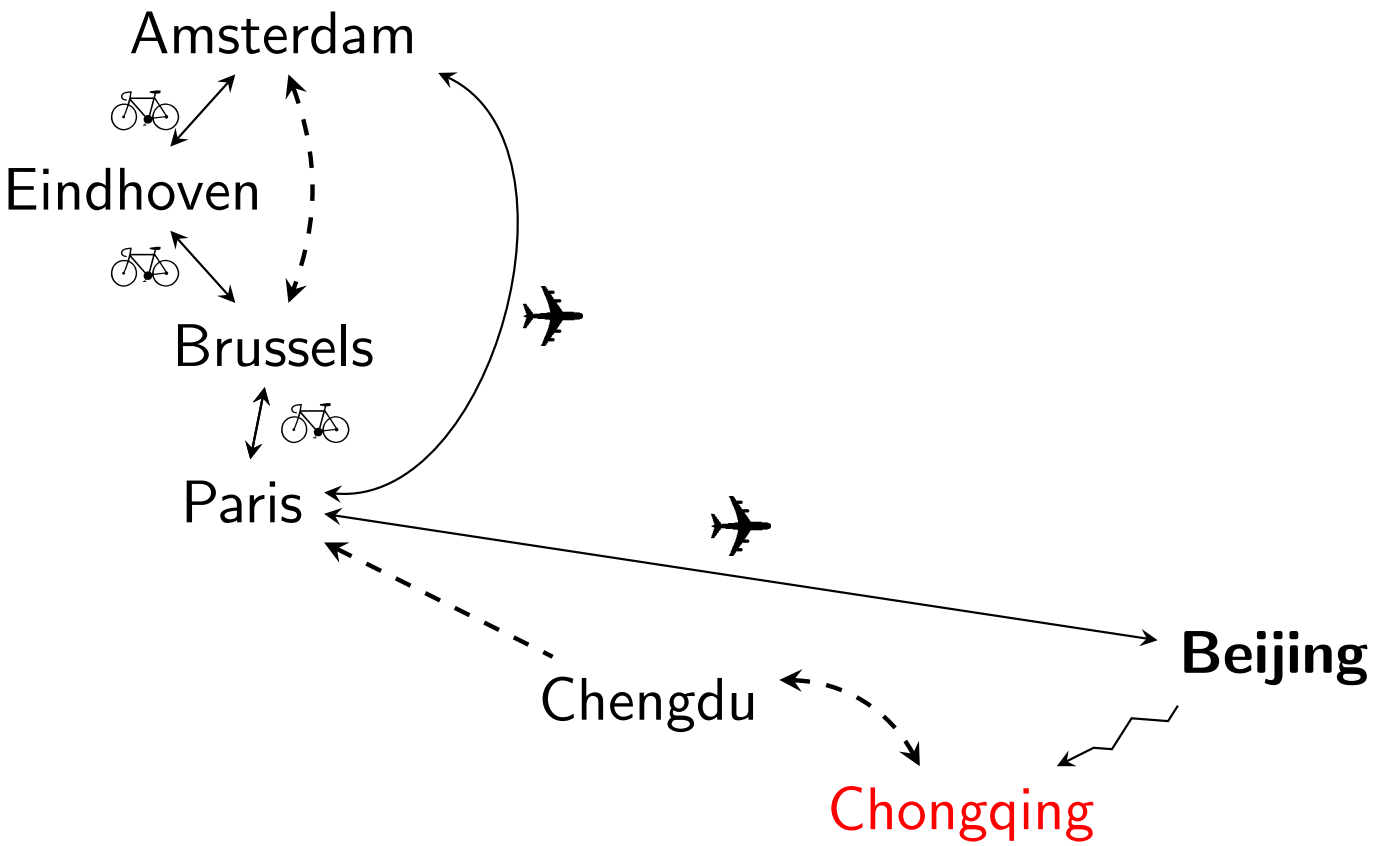

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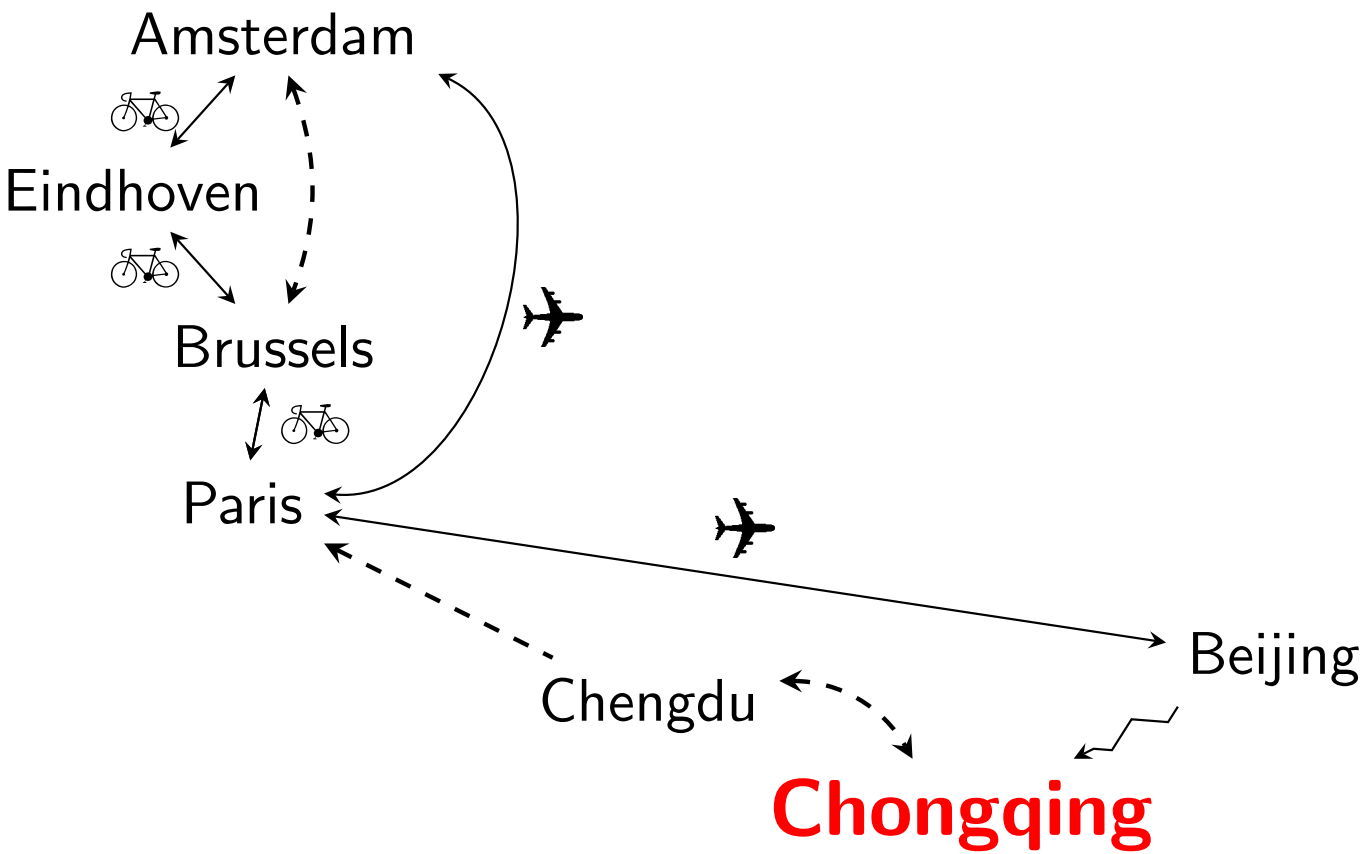


Sabotage Modal Logic

Sabotage Model

Take a finite Σ. The model $M = \langle W, (R_{a_i})_{a_i \in \Sigma}, Val \rangle$ is given by

$$W \neq \emptyset, \qquad R_{a_i} \subseteq W \times W, \qquad Val : \texttt{PROP} \to \mathcal{P}(W)$$

Removal Operation

Let $M = \langle W, \{R_a \mid a \in \Sigma\}, Val \rangle$ be a Sabotage Model.

$$M^{a_i}_{(v,v')} := \langle W, R_{a_1}, \ldots R_{a_{i-1}}, R_{a_i} \setminus \{(v, v')\}, R_{a_{i+1}}, \ldots R_{a_n}, Val \rangle$$

Sabotage Modal Logic

Sabotage Modal Language

Language: $\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \Diamond_a \varphi \mid \diamondminus_a \varphi \qquad p \in \mathtt{PROP},\ a \in \Sigma$

Abbreviations: $\Diamond\varphi := \bigvee_{a\in\Sigma} \Diamond_a \varphi \qquad \diamondminus\varphi := \bigvee_{a\in\Sigma} \diamondminus_a \varphi$

Semantics

$$M, w \models \diamondminus_a \varphi \quad \text{iff} \quad \text{there is } (u, v) \in R_a \text{ s. t. } M^a_{(u,v)}, w \models \varphi$$

Theorem

Model checking of SML is PSPACE-complete (combined compl.).

Sabotage Learning Game

Definition

A *Sabotage Learning Game* is a Sabotage Game played between *Learner* and *Teacher* on a directed multi-graph with an initial vertex and a "goal" vertex.

Various scenarios

Game	Winning Condition
SLG_{UE} unhelpful T., eager L.	Learner wins iff he reaches the goal state, Teacher wins otherwise.
SLG_{HU} helpful T., unwilling L.	Teacher wins iff Learner reaches the goal state, Learner wins otherwise.
SLG_{HE} helpful T. eager L.	Both players win iff Learner reaches the goal state, Both lose otherwise.

Characterization Results

Theorem

Game	Existence of winning strategy	Winner
SLG_{UE}	$\gamma_0^{UE} := goal,$	*Learner*
unhelpful T., eager L.	$\gamma_{n+1}^{UE} := goal \vee \Diamond \boxminus \gamma_n^{UE}$	
SLG_{HU}	$\gamma_0^{HU} := goal,$	*Teacher*
helpful T., unwilling L.	$\gamma_{n+1}^{HU} := goal \vee (\Diamond \top \wedge (\Box \diamondminus \gamma_n^{HU}))$	
SLG_{HE}	$\gamma_0^{HE} := goal,$	*Both*
helpful T. eager L.	$\gamma_{n+1}^{HE} := goal \vee \Diamond \diamondminus \gamma_n^{HE}$	

Complexity of Sabotage-Type Learning

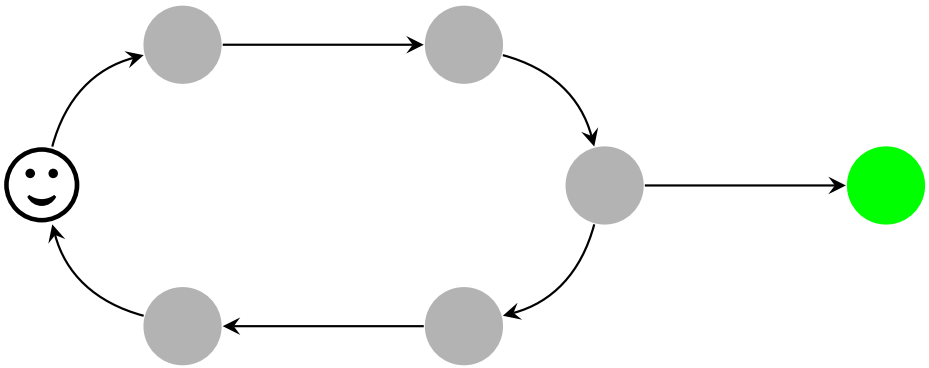
Theorem

Game	Winning Condition	Complexity
SLG_{UE}	*Learner wins iff he reaches the goal state, Teacher wins otherwise*	*PSPACE-complete*
SLG_{HU}	*Teacher wins iff Learner reaches the goal state, Learner wins otherwise.*	*PSPACE-complete*
SLG_{HE}	*Both players win iff Learner reaches the goal state. Both lose otherwise.*	*NL-complete*

Local vs global moves

- Players' moves are of a different nature:
  - Learner moves by *local* transitions.
  - Teacher moves by *globally* removing an edge.
- Teacher only needs to act when necessary.

Non-strict alternation – Example



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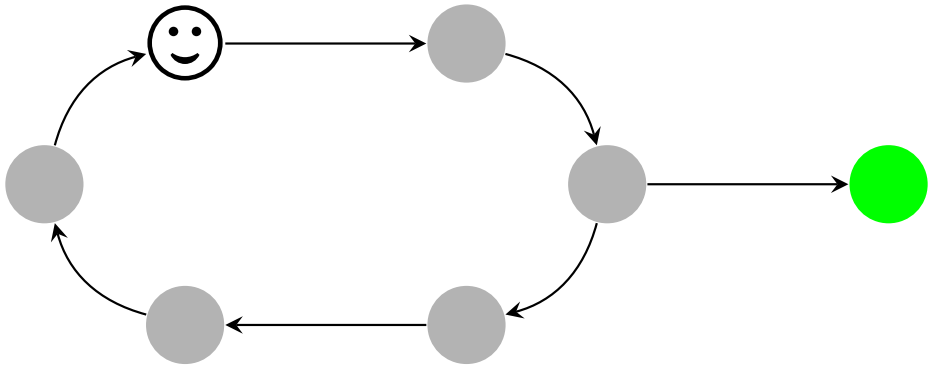

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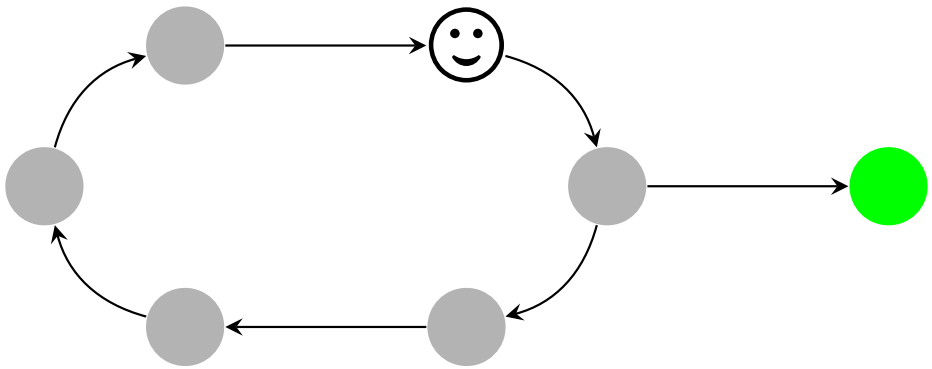

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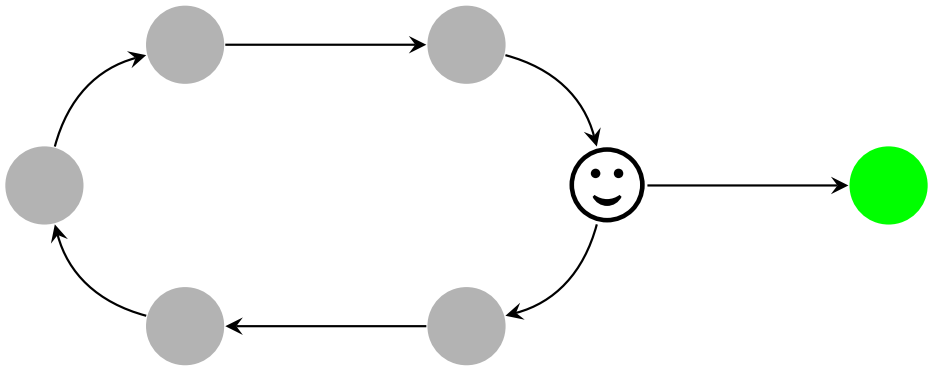

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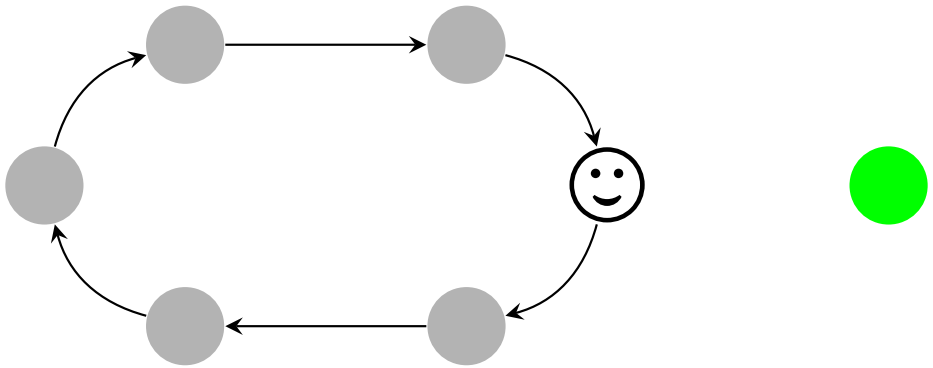

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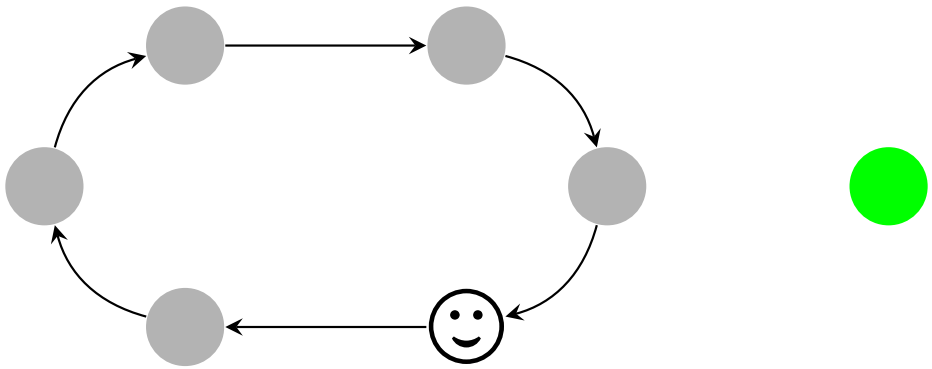

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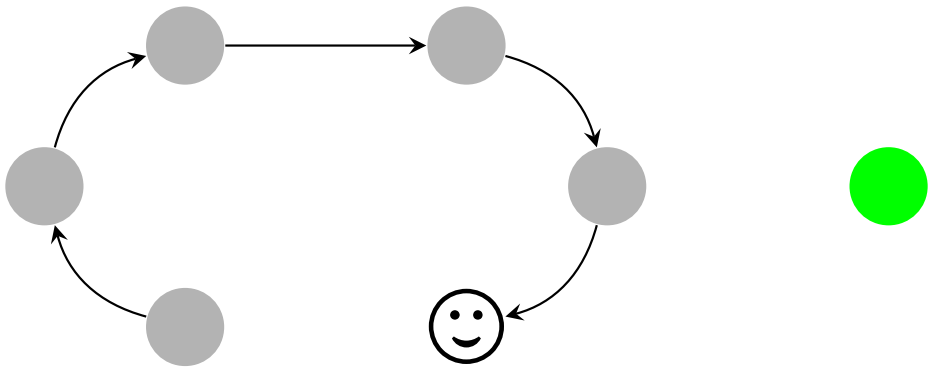

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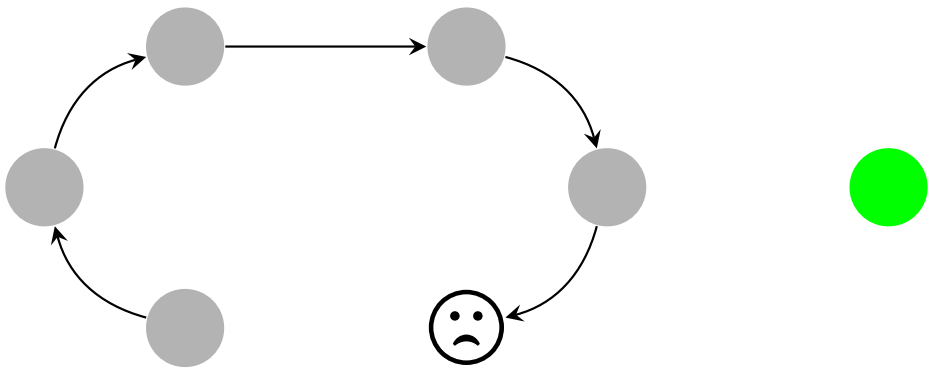

Different game, same scenarios

Game	Winning Condition
SLG^*_{UE} unhelpful T., eager L.	Learner wins iff he reaches the goal state, Teacher wins otherwise.
SLG^*_{HU} helpful T., unwilling L.	Teacher wins iff Learner reaches the goal state, Learner wins otherwise.
SLG^*_{HE} helpful T. eager L.	Both players win iff Learner reaches the goal state, Both lose otherwise.

Strict vs non-strict alternation

Theorem

1. *Learner has a w.s. in* SLG^*_{UE} *iff he has a w.s. in* SLG_{UE}.
2. *Teacher has a w.s. in* SLG^*_{HU} *iff she has a w.s. in* SLG_{HU}.
3. *Teacher and Learner have a joint w.s. in* SLG^*_{HE} *iff they have a joint w.s. in* SLG_{HE}.

Corollary

- SLG^*: same complexity and characterization results as SLG.

Summary & Conclusion

Aim

High-level GT perspective on formal learning theory.
Strategic abilities, information flow and interaction.

Summary & Conclusion

- GT approach to learning that accounts for different levels of cooperativeness between Learner and Teacher.
- Syntactic characterization and complexity of the variations.
- Deciding whether Teacher can force the unwilling Learner to learn is as difficult as deciding whether Learner can learn when Teacher is unhelpful
- Teacher's "break" does not change ability to win.

Further Work

- Other variations on sabotage:
  - local sabotage
  - adding transitions ($\rightarrow$ reactive Kripke structures)
  - sabotage game with imperfect information
- Identification in the limit (stable positions).
  - Epistemic and doxastic interpretation.
  - Fixed-point logics.
- GT approach to learning algorithms.